

Exercise- 10.3

Q. (7). $(3\vec{a} - 5\vec{b}) \cdot (2\vec{a} + 7\vec{b})$ पृथक्फलको डीलिट्रार (find the product)

Solⁿ

$$\begin{aligned} & (3\vec{a} - 5\vec{b}) \cdot (2\vec{a} + 7\vec{b}) \\ &= (3\vec{a}) \cdot (2\vec{a}) + (3\vec{a}) \cdot (7\vec{b}) + (5\vec{b}) \cdot (2\vec{a}) - (5\vec{b}) \cdot (7\vec{b}) \\ &= 6(\vec{a} \cdot \vec{a}) + 21\vec{a} \cdot \vec{b} - 10\vec{b} \cdot \vec{a} - 35(\vec{b} \cdot \vec{b}) \\ &= 6a^2 + 21\vec{a} \cdot \vec{b} - 10\vec{a} \cdot \vec{b} - 35b^2 \quad [\because \vec{a} \cdot \vec{a} = a^2, \vec{b} \cdot \vec{b} = b^2] \\ &= 6a^2 + 11\vec{a} \cdot \vec{b} - 35b^2, \quad \text{where } |\vec{a}| = a \\ & \qquad \qquad \qquad |\vec{b}| = b. \end{aligned}$$

Q. (8). \vec{a} आ \vec{b} डेकोर डूरुओ खान एक, डिस्टेंड खानुओ कोनको 60° आ डिस्टेंड खलर पृथक्फल $\frac{1}{2}$. डेकोर डूरुओ खान डीलिट्रार ।

Solⁿ धरौं, \vec{a} आ \vec{b} डेकोर खान θ
(let the angle betⁿ \vec{a} and \vec{b} be θ).

Now, we know,

$$\vec{a} \cdot \vec{b} = |\vec{a}| |\vec{b}| \cos \theta$$

$$\Rightarrow \frac{1}{2} = |\vec{a}| |\vec{b}| \cos 60^\circ, \quad \because \text{दिअ खानुओ (given)}$$

$\vec{a} \cdot \vec{b} = \frac{1}{2}$,
 $|\vec{a}| = |\vec{b}|$

$$\Rightarrow \frac{1}{2} = |\vec{a}|^2 \cdot \frac{1}{2}$$

$$\Rightarrow |\vec{a}|^2 = 1$$

$$\Rightarrow |\vec{a}| = 1$$

Thus, $\left\{ \begin{array}{l} |\vec{a}| = 1 \\ |\vec{b}| = 1 \end{array} \right\} \leftarrow \text{Ans.}$

Q. 6) ଏଠାରେ ଏକକ ସଦ୍ୱେକ \hat{a} ଓ \hat{b} ଯାହା $(\vec{n} - \hat{a}) \cdot (\vec{n} + \hat{a}) = 12$ ଓ $|\vec{n}|$ ନିର୍ଣ୍ଣୟ କର ।

Solⁿ

we have,

$$(\vec{n} - \hat{a}) \cdot (\vec{n} + \hat{a}) = 12$$

$$\Rightarrow \vec{n} \cdot \vec{n} + \vec{n} \cdot \hat{a} - \hat{a} \cdot \vec{n} - \hat{a} \cdot \hat{a} = 12$$

$$\Rightarrow |\vec{n}|^2 - 1 = 12,$$

$$\Rightarrow |\vec{n}|^2 = 13$$

$$\Rightarrow |\vec{n}| = \sqrt{13} \leftarrow \text{Ans.}$$

$$\vec{n} \cdot \vec{n} = |\vec{n}|^2$$

$$\vec{n} \cdot \hat{a} = \hat{a} \cdot \vec{n}$$

$$\hat{a} \cdot \hat{a} = |\hat{a}| |\hat{a}| \cos 0^\circ$$

$$= 1 \cdot 1 \cdot 1$$

$$= 1$$

Q. No. 10.

$$\vec{a} = 2\hat{i} + 2\hat{j} + 3\hat{k}, \quad \vec{b} = -\hat{i} + 2\hat{j} + k \quad \text{and} \quad \vec{c} = 3\hat{i} + \hat{j}$$

তিনটি ভেক্টর \vec{a} ও \vec{b} এর $\vec{a} + \lambda\vec{b}$, \vec{c} এর উপর লম্বা, λ -এর মান নির্ণয় কর।

Solⁿ We have

$$\begin{aligned} \vec{a} + \lambda\vec{b} &= (2\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(-\hat{i} + 2\hat{j} + k) \\ &= (2-\lambda)\hat{i} + (2+2\lambda)\hat{j} + (3+\lambda)\hat{k} \end{aligned}$$

Now, given, $\vec{a} + \lambda\vec{b}$ ভেক্টরটি \vec{c} -এর উপর লম্বা।
($\vec{a} + \lambda\vec{b} \perp \vec{c}$)

$$\Rightarrow (\vec{a} + \lambda\vec{b}) \cdot \vec{c} = 0$$

$$\Rightarrow \{(2-\lambda)\hat{i} + (2+2\lambda)\hat{j} + (3+\lambda)\hat{k}\} \cdot (3\hat{i} + \hat{j}) = 0$$

$$\Rightarrow (2-\lambda) \cdot 3 + (2+2\lambda) \cdot 1 + (3+\lambda) \cdot 0 = 0$$

$$\Rightarrow 6 - 3\lambda + 2 + 2\lambda = 0$$

$$\Rightarrow 8 - \lambda = 0$$

$$\Rightarrow \lambda = 8 \leftarrow \text{Ans.}$$

=====

$$\vec{a} \perp \vec{b} \Leftrightarrow \vec{a} \cdot \vec{b} = 0$$