For material information, in general co-ordinate system, another important function of the matric tensor is to relate the covariant and contravariant components of a base vector within the given co-ordinate system.

Suppose,

$$g_{jk} = \text{pairwise} = g_{kj} - \text{Symmetric tensor}$$

$$g_{jk} = -g_{kj} \quad - \text{anti-symmetric tensor.}$$

Co-ordinate systems, Base vectors, Covariant & Contravariant

Let ~~two~~ a nonorthogonal co-ordinate system (eg-crystallographic) having ~~base~~ base vectors with destinct magnitudes usualy bent or curved,

$$A = a_x i + a_y j + a_z k$$

a_x, a_y & a_z are the component of A along i, j & k that seperated $XY(12)$ $XZ(13)$ & $YZ(23)$ plane.

Considering a point P away from origin in space with a vector V having local basis, we may specify our local basis at P in one of two ways.

(1) setting local axes at P using local co-ordinate curve which parallel to Cartesian co-ordinate axes. Then ~~choose~~ choose a set tangent to each of the local axes at P and call items as e_1, e_2 & e_3 and V specify as a

a linear Combination of these three vectors. Known as contravariant components of the vectors.

② Setting another three sets of base vectors which are e_1^*, e_2^* & e_3^*; mutually perpendicular to each other. The linear combination these three ~~comb~~ base vectors ~~const~~ constitute the vector V, to the local cordinate systems; known as covariant components of the vectors omitting i, j & K.

~~Since Eucleidean geomatry that two non-parallel with the combination points~~

Since Eucledean geomatry two non parallel intersecting lines constitute a plane and each product with co-ordinate pair.

Space collections of points may be the n-dimensional & m-dimensional space may used to determine a new unique $(m+n)$ dimensional product space. The product space it may be result of lines or curves. i.e →

① Contravariant base vectors related to the curve, usually denoted by superscripts i.e

$\{e^1, e^2\ e^3\}$ → for K system

$\{e^{1*}\ e^{2*}\ e^{3*}\}$ — for K^* system

② Covariant base vectors related to the lines usually denoted by subscript

$\{e_1, e_2, e_3\} \longrightarrow$ for K system

$(e_1^*, e_2^*, e_3^*) \longrightarrow$ for K^* "

1. The vector v in its contravariant and covariant components is

$$V = V^1 e^1 + V^2 e^2 + V^3 e^3 = V_1 e_1 + V_2 e_2 + V_3 e_3$$

$$V^* = V^{1*} e^{1*} + V^{2*} e^{2*} + V^{3*} e^{3*} = V_1^* e_1^* + V_2^* e_2^* + V_3^* e_3^*$$

2. For dyad: $\underline{G}$

(a) Covariant $g_{jk} = e^j \cdot e^k$ | j & k are 1, 2, 3 - -

(b) Contravariant $g^{jk} = e_j \cdot e_k$

---

x-axis	1 axis	plane yz	→ 23 plane
y "	2 "	" xz	— 13
z "	3 "	" xy	— 12

so, $e_x = e_1$ $e_y = e_2$ $e_z = e_3$

e

Lect-III

Kronecker Delta and Identity Matrix :-

All these information ~~can~~ about contravariant and covariant base of basis vectors may be summerised single equation. Let we must first introduce a perticular symbol named as Kroneker's Delta

(Leopold Kronecker, a German mathematician)

ie $\delta^j_k \rightarrow$ appears to mix covariant and contravariant indices.

$$\delta^j_k = 1 \quad \text{when} \quad j = k$$

$$\delta^j_k = 0 \quad \text{when} \quad j \neq k$$

Now summerise the relationship betn contravariant and covariant base vectors as

$$e^j \cdot e_k = \delta^j_k = e_k \cdot e^j = \delta^k_j$$

Thus $\delta^1_1 = 1$ $\quad \delta^2_1 = 0$ $\quad \delta^3_1 = 0$

$\delta^2_1 = 0$ $\quad \delta^2_2 = 1$ $\quad \delta^3_2 = 0$

$\delta^3_1 = 0$ $\quad \delta^3_2 = 0$ $\quad \delta^3_3 = 1$

Thus square matrix $\underline{I}$

$$\underline{I} = \begin{vmatrix} \delta^1_1 & \delta^2_1 & \delta^3_1 \\ \delta^2_1 & \delta^2_2 & \delta^3_2 \\ \delta^3_1 & \delta^3_2 & \delta^3_3 \end{vmatrix}$$

For any n-ads $\underline{X}$

$$\underline{I} \cdot \underline{X} = \underline{X} \cdot \underline{I} = \underline{X}$$

Dyad Components.

(Covariant, Contravariant, Mixed dyad)

(1) For typical dyads

$$\underline{D} = A B$$

The vectors A & B may have individually

(a) Covariant – Covariant

(b) Contravariant – Contravariant

(c) Covariant – Contravariant

(d) Contravariant – Covariant

ie

(1) Covariant $a_j b_k = c_{jk}$

(2) Mixed — $a_j b^k = c_j^{\ k}$

(3) Mixed $a^j b_k = c_k^{\ j}$

(4) Contravariant. $a^j b^k = c^{jk}$

Relationship between — covariant and contravariant of a vectors :-

Let $v = v^1 e^1 + v^2 e^2 + v^3 e^3$

$= v_1 e_1 + v_2 e_2 + v_3 e_3$

ie $e^j \cdot e^k = g_{jk}$

$e_j \cdot e_k = g^{jk}$

$e^j \cdot e_k = e_k \cdot e^j = \delta^k_j$

To find relationships betn two sets of vectors Component.

Taking inner product

$v \cdot e^1 = (v^1 e^1 + v^2 e^2 + v^3 e^3) \cdot e^1$

$= (v_1 e_1 + v_2 e_2 + v_3 e_3) \cdot e_1$

ie $v_1 = g_{11} v^1 + g_{12} v^2 + g_{13} v^3$

$v_2 = g_{21} v^1 + g_{22} v^2 + g_{23} v^3$

$v_3 = g_{31} v^1 + g_{32} v^2 + g_{33} v^3$

&

$v^1 = g^{11} v_1 + g^{12} v_2 + g^{13} v_3$

$v^2 = g^{21} v_1 + g^{22} v_2 + g^{23} v_3$

$v^3 = g^{31} v_1 + g^{32} v_2 + g^{33} v_3$

Let

$$V_c = \begin{vmatrix} v_1 \\ v_2 \\ v_3 \end{vmatrix} \quad \& \quad V^c = \begin{vmatrix} v^1 \\ v^2 \\ v^3 \end{vmatrix}$$

$$\underline{G_c} = \begin{vmatrix} g_{11} & g_{12} & g_{13} \\ g_{21} & g_{22} & g_{23} \\ g_{31} & g_{32} & g_{33} \end{vmatrix}$$ — Covariant dyad $\underline{G_c}$

$$G^c = \begin{vmatrix} g^{11} & g^{12} & g^{13} \\ g^{21} & g^{22} & g^{23} \\ g^{31} & g^{32} & g^{33} \end{vmatrix}$$ contravariant dyad $\underline{G^c}$

Hence $V_c = G_c V^c$ and $V^c = G^c V_c$

Here we may write

$$v_j = \Sigma_k \, g_{jk} v^k \quad \text{and} \quad v^j = \Sigma_k \, g^{jk} v_k.$$

** Einstein noticed that the summation sign Σ_K was redundant in these equations and all other repeated index. In each case above summation is occurring over the index K, which is repeated once as a convariant index and once as a contravariant index in each terms: Thus finally we write the tensor analysis.

$$v_j = g_{jk} v^k \quad \& \quad v^j = g^{jk} v_k.$$

This convention is called Einstein Summation