$$n' = \left(n - \frac{1}{t}\right)$$

Hence the correct value of the frequency of unknown tuning fork can be determined.

1.20 Beats

When two waves trains, slightly differing in frequencies (e.g., from two tuning forks of nearly equal frequencies) travel along the same straight line in the same direction, then the resultant amplitude is alternately maximum and minimum. Thus the intensity of sound, which is proportional to the square of amplitude rises and falls (technically known as waxing and waning of sound) alternately with time. This phenomenon of waxing and waning of sound is called beats. The number of waxing and waning in one second is called the frequency of beasts. This frequency of beats is equal to the difference in the frequencies of the sound waves.

Production of beats

The phenomenon of beats occurs as a result of interference between two sound waves of slightly different frequencies travelling along the same straight line in the same straight line in the same direction. Consider that a particular instant t_1 (Fig 1.12), the two waves meet in the same phase at a particular point. They reinforce to produce maximum sound intensity. After this instant, they get further and further out of phase as their frequencies are slightly different. After a short time (at time t_2) the two waves arrive at the point in the opposite phase. This happens when one wave gains half a vibration on the other. Now they produce minimum sound intensity. Again after some time i.e., at instant t_3 one wave gains one full vibration on the other and the two waves are again in phase and produce maximum and one minimum constitute on beat. The number of beats per sec. is equal to the difference in frequencies of the sources.

Now we shall explain the production of beats by considering the case of two tuning forks of frequencies 256 and 254. Let the two forks start vibrating together in the same phase. After 1/4 second, the first fork completes its 64 vibrations while the second one has completed its $63_{1/2}$ vibrations. The two waves are now in opposite phase and produce minimum intensity. After 1/2 second, the two waves are again in phase (phase difference is equal to λ) and produce maximum intensity. After 3/4 second, the first

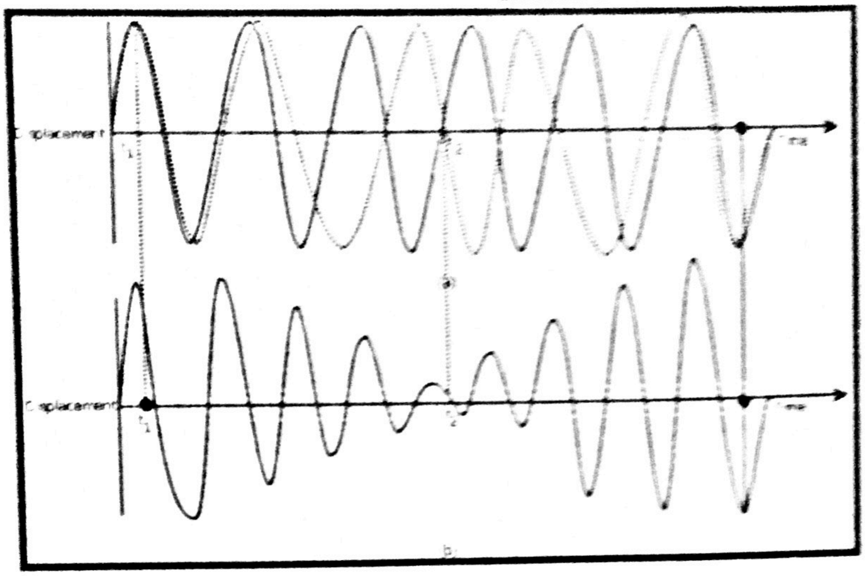

Figure 1.12

fork completes 192 vibrations while the second one completes 192 1/2 vibrations. There is phase change of 3λ/2 i.e., the two waves are in opposite phase and produce minimum intensity. After completion of one second, they are gain in phase and produce maximum sound intensity. During one sec. two maxima and two minima are recorded i.e., two beats are heard in one sec. Hence the number of beats equal to the difference in the frequencies of the two sources.

Mathematical analysis. Consider the case of two waves having same amplitude a with slightly different frequencies n_1 and n_2 travelling simultaneously in medium. If y_1 and y_2 be the displacements of these waves at any instant t, then

$$y_1 = a \sin 2\pi n_1 t \quad \text{...(1)}$$

and

$$y_2 = a \sin 2\pi n_2 t \quad \text{...(2)}$$

Applying the principle of superposition, the resultant displacement y at any instant t is given by,

$$y = y_1 + y_2 = a \sin 2\pi n_1 t + a \sin 2\pi n_2 t$$

$$= A \sin 2\pi \left(\frac{n_1 + n_2}{2}\right) t \quad \text{...(3)}$$

$$\text{where } A = 2a \cos 2\pi \left(\frac{n_1 - n_2}{2}\right) t$$

Equation (3) represents a simple harmonic motion whose amplitude

$$A = 2\,a\,\cos 2\pi\left(\frac{n_1 - n_2}{2}\right)t$$

and frequency = $(n_1+n_2)/2$

Now for A to be maximum,

$$= \cos 2\,\pi\left(\frac{n_1 - n_2}{2}\right)t = \pm 1$$

$$or\;\; 2\pi\left(\frac{n_1 - n_2}{2}\right)t = k\pi \;where\; k = 0, 1, 2, \ldots$$

$$or\; t = \frac{k}{(n_1 - n_2)}$$

when k = 0, t = 0 First maxima

$$k = 1, t_1 = \frac{1}{(n_1 - n_2)}$$ second maxima

$$k = 2, t_2 = \frac{2}{(n_1 - n_2)}$$ Third maxima

$$k = n, t_n = \frac{n}{(n_1 - n_2)}\, nth\ maxima$$

Thus the time interval between two successive maxima

$$= \frac{1}{(n_1 - n_2)}\, seconds$$

$$\therefore Frequency\ of\ maxima = (n_1 - n_2)$$

Similarly, the amplitude is minimum, when

$$\cos 2\pi\left(\frac{n_1 - n_2}{2}\right)t = 0$$

or $2\pi\left(\frac{n_1 - n_2}{2}\right)t = (2k+1)\pi/2$

where k = 0, 1, 2, 3, ...

or $t = \frac{(2k+1)}{2(n_1 - n_2)}$

where $k = 0, t = \frac{1}{2(n_1 - n_2)}$

$k = 1, t_1 = \frac{3}{2(n_1 - n_2)}$

$k = n, t_n = \frac{(n+1)}{2(n_1 - n_2)}$

The interval between two successive minima

$= \frac{1}{(n_1 - n_2)}\text{sec}$

$\therefore$ Frequency of minima $= (n_1 - n_2)$

This shows that the frequency of minima is the same as that of maxima. In one sec. the intensity is maximum (n_1-n_2) time and minimum also (n_1-n_2) times. Hence (n_1-n_2) beats will be heard in one second.

Difference between interference and beats.

When two sound waves of exactly equal frequencies are superimposed over each other the redistribution of energy takes place and the phenomenon is known as interference. In this case the intensity of sound at any point remains constants for all times i.e., the positions of maxima and minima of sound do not shift with time. Thus, the interference pattern remains fixed in space. On the other hand, when the sound waves of slightly different frequencies are superimposed over each other beats are produced. The intensity of sound at any point alternately rises and falls i.e., the position of maxima and minima change with time. Thus the interference pattern does not remain stationary.

(1) In case of interference the two frequencies should be exactly same while in case o beats it changes from point to point.

(2) In case of interference, the intensity of sound at any particular point is constan while in case of beats it changes from point to point.

Determination of frequency of tuning fork by the method of beats.

In order to determine the frequency of a given tuning fork, say A, it is sounded with tuning fork, say B, of known frequency n.

Now the number of beats produced in a given interval of the time is counted. Let x b the number of beats heard per second, then the frequency of A will be either (n+x) o (n-x). To decide which of the two values is correct, the prong of unknown fork A i loaded with little wax. They are again sounded together and number of beats i counted. If the tuning fork A has a higher frequency i.e., its frequency will be (n+x and if number of beats increases then the frequency of A will be (n-x). In this way th unknown frequency can be determined.

MULTIPLE CHOICE QUESTIONS

1. The interference phenomenon can take place:

a. In all waves

b. In transverse waves only

c. In longitudinal waves only

d. In standing waves only

2. A particle moves in x-y plane according to the equation of motion of the particle is

a. On a straight line

b. On an ellipse

c. Periodic

d. Simple Harmonic