Exercise 10.3

Q.2. $\hat{i}-2\hat{j}+3\hat{k}$ এবং $3\hat{i}-2\hat{j}+\hat{k}$ ভেক্টৰ দুটাৰ মাজৰ কোণ (angle) উলিওৱা

সমাঃ Let

$$\vec{a} = \hat{i}-2\hat{j}+3\hat{k}$$

$$\vec{b} = 3\hat{i}-2\hat{j}+\hat{k}$$

Let angle between $\vec{a}$ and $\vec{b}$ is θ.

$$\therefore \quad \vec{a}\cdot\vec{b} = |\vec{a}|\,|\vec{b}|\cos\theta$$

$$\Rightarrow \boxed{\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|\,|\vec{b}|}} \longrightarrow (1)$$

Now,

$$\vec{a}\cdot\vec{b} = (\hat{i}-2\hat{j}+3\hat{k})\cdot(3\hat{i}-2\hat{j}+\hat{k})$$

$$= (1\cdot 3) + \{-2\cdot(-2)\} + 3\cdot 1$$

$$= 3 + 4 + 3 = 10.$$

$$|\vec{a}| = \sqrt{1^2+(-2)^2+3^2} = \sqrt{1+4+9} = \sqrt{14}$$

$$|\vec{b}| = \sqrt{3^2+(-2)^2+1^2} = \sqrt{9+4+1} = \sqrt{14}$$

$$\therefore \cos\theta = \frac{10}{\sqrt{14}\,\sqrt{14}} = \frac{10}{14} = \frac{5}{7}$$

$$\therefore \theta = \cos^{-1}\left(\frac{5}{7}\right) \longleftarrow \text{Ans.}$$

Exercise 10.3

Q. 5 দেখুওৱা যে অধোলিখিত ভেক্টৰ তিনিটাৰ প্ৰতিটোৱেই একক ভেক্টৰ
(Prove that the following three vectors are unit vector)

$$\frac{1}{7}(2\hat{i}+3\hat{j}+6\hat{k}),\ \frac{1}{7}(3\hat{i}-6\hat{j}+2\hat{k}),\ \frac{1}{7}(6\hat{i}+2\hat{j}-3\hat{k})$$

আকৌ দেখুওৱা যে ইহঁত পৰস্পৰ লম্ব।
(Again show that They are mutually perpendicular)

উঃ Let,

$$\vec{a} = \frac{1}{7}(2\hat{i}+3\hat{j}+6\hat{k})$$

$$\vec{b} = \frac{1}{7}(3\hat{i}-6\hat{j}+2\hat{k})$$

$$\vec{c} = \frac{1}{7}(6\hat{i}+2\hat{j}-3\hat{k}).$$

Now,

$$|\vec{a}| = \left|\frac{1}{7}\right|\sqrt{2^2+3^2+6^2} = \frac{1}{7}\sqrt{4+9+36} = \frac{1}{7}\cdot\sqrt{49} = \frac{1}{7}\cdot 7 = 1$$

$$|\vec{b}| = \left|\frac{1}{7}\right|\sqrt{3^2+(-6)^2+2^2} = \frac{1}{7}\sqrt{49} = \frac{1}{7}\cdot 7 = 1$$

$$|\vec{c}| = \left|\frac{1}{7}\right|\sqrt{6^2+2^2+(-3)^2} = \frac{1}{7}\sqrt{49} = \frac{1}{7}\cdot 7 = 1$$

$\therefore$ $\vec{a}, \vec{b}, \vec{c}$ ভেক্টৰ তিনিটা একক ভেক্টৰ (unit vectors).

Again,

$$\vec{a}\cdot\vec{b} = \frac{1}{7}\cdot\frac{1}{7}(2\cdot 3+3\cdot(-6)+6\cdot 2) = \frac{1}{49}\cdot(0) = 0$$

$$\vec{b}\cdot\vec{c} = \frac{1}{7}\cdot\frac{1}{7}(3\cdot 6+(-6)\cdot 2+2\cdot(-3)) = \frac{1}{49}\cdot(0) = 0$$

$$\vec{c}\cdot\vec{a} = \frac{1}{7}\cdot\frac{1}{7}(6\cdot 2+2\cdot 3+(-3)\cdot 6) = \frac{1}{49}\cdot(0) = 0.$$

Hence $\vec{a}, \vec{b}, \vec{c}$ are mutually $\perp^r$ ($\vec{a}, \vec{b}, \vec{c}$ পৰস্পৰ লম্ব)।

Exercise 10.3

Q.6. যদি $(\vec{a}+\vec{b})\cdot(\vec{a}-\vec{b}) = 8$ আৰু $|\vec{a}| = 8|\vec{b}|$, তেন্তে $|\vec{a}|$ আৰু $|\vec{b}|$ উলিওৱা।

[If $(\vec{a}+\vec{b})\cdot(\vec{a}-\vec{b}) = 8$ and $|\vec{a}| = 8|\vec{b}|$, then find $|\vec{a}|$ and $|\vec{b}|$].

Sol^n. দিয়া আছে (Given),

$$(\vec{a}+\vec{b})\cdot(\vec{a}-\vec{b}) = 8$$

$$\Rightarrow \vec{a}\cdot\vec{a} - \vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{a} - \vec{b}\cdot\vec{b} = 8$$

$$\Rightarrow \vec{a}^2 - \vec{b}^2 = 8, \quad \because \vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{a}$$

$$\Rightarrow |\vec{a}|^2 - |\vec{b}|^2 = 8, \quad \because \vec{a}\cdot\vec{a} = \vec{a}^2 = |\vec{a}|^2$$

$$\Rightarrow \{8|\vec{b}|\}^2 - |\vec{b}|^2 = 8$$

$$\Rightarrow 64|\vec{b}|^2 - |\vec{b}|^2 = 8$$

$$\Rightarrow 63|\vec{b}|^2 = 8$$

$$\Rightarrow |\vec{b}|^2 = \frac{8}{63}$$

$$\Rightarrow |\vec{b}| = \sqrt{\frac{8}{63}} = \frac{\sqrt{8}}{\sqrt{63}} = \frac{2\sqrt{2}}{3\sqrt{7}}$$

$$\therefore |\vec{a}| = 8|\vec{b}| \Rightarrow \vec{a} = 8 \times \frac{2\sqrt{2}}{3\sqrt{7}} = \frac{16\sqrt{2}}{3\sqrt{7}}$$

Thus, $|\vec{a}| = \frac{2\sqrt{2}}{3\sqrt{7}}$

& $|\vec{b}| = \frac{16\sqrt{2}}{3\sqrt{7}}$ ← **Ans.**