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Some definitions:

Atomic mass unit:- one atomic mass unit (a.m.u or 1u) is equal to 1/12 of the mass of neutral carbon atom taken as 12 (C_6^{12}).

$$1\,amu = 1.6604 \times 10^{-27}\,kg.$$

Taking the velocity of light $c = 2.998 \times 10^{8}$ m/sec and

$$1\,Joul = 1.602 \times 10^{13}\,MeV$$

we have $1\,amu = 1.6604 \times 10^{-27} \times (2.998 \times 10^{8})^2$ ~~$\times 2.998 \times 10^{16}$~~

$$= 14.92 \times 10^{-11}\,Joules. \quad \because m = m_0c^2$$

$$= \frac{14.92 \times 10^{-11}}{1.602 \times 10^{13}} = 931.5\,MeV$$

Mass defect:- The nucleus is formed by bringing protons and neutrons together. The mass of the nucleus so formed is less than the sum of the masses of the constituent protons and neutrons. This mass difference is called mass defect and is denoted by Δm.

If Z is the number of protons in the nucleus (atomic number Z), then the number of neutrons in the nucleus is $(A-Z)$ (A is the atomic mass). If m_p is the mass of the proton and m_n is that of neutron then sum of the masses of proton and neutrons

$$Zm_p + (A-Z)m_n$$

If M_N is the actual mass of the nucleus then

$$\text{mass defect } \Delta m = Zm_p + (A-Z)m_n - M_N$$

mass defect is therefore defined as the difference between the sum of the rest masses of the nucleons forming the nucleus and the actual rest mass of the nucleus.

For example in the case of deuteron which contains one proton and one neutron, the combined mass is $(1.0073 + 1.0087) = 2.1060$ amu, whereas the actual mass of deuteron nucleus is 2.0136 amu.

$$\text{mass defect } \Delta m = 2.1060 - 2.0136 = 0.0024\,amu$$

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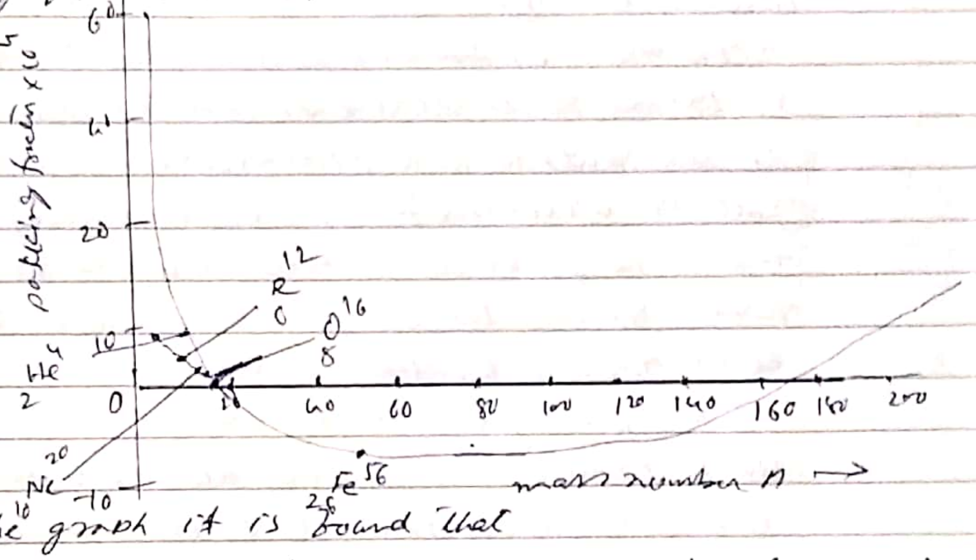
Packing Fraction :- It has been found that atomic masses though very close to whole number values invariably differ from the integral value by a small amount. For example the atomic masses of H^1 He^4 and Li^6 are $H^1 = 1.007825$, $He^4 = 4.002603$, $Li^6 = 6.015126$

The deviation of atomic mass from whole number value is expressed in the form of a quantity known as packing fraction. Packing fraction is defined as the ratio of the difference between the atomic mass of the atom M and its mass number A (i.e. M−A) to its mass number A.

$\therefore$ packing fraction, $f = \frac{M-A}{A}$.

Variation of packing fraction (multiplied by a factor 10^4) and mass number A is shown in the fig below. It is known as packing fraction curve.



From the graph it is found that

(i) For very light nuclei, the packing fraction f is maximum indicating thereby that these nuclei are unstable.

(ii) As the value of A increases, packing fraction goes on decreasing with exception of $^{4}_{2}He$ and $^{12}_{6}C$ till it becomes zero for A = 16. Thus the packing fraction for $^{16}_{8}O$ is zero.

(iii) As the mass number A increases beyond 16 the values of f goes on decreasing further and becomes negative. This continues approximately up to A = 180 (Ta, Tantalum) beyond which f again becomes positive.

(iv) The value of f is minimum for iron ($^{56}_{26}Fe$) which has a significance as the most stable nucleus.

(V) Greater part of the curve is below zero line with negative values of packing fraction indicating that there is loss of mass and hence liberation of energy due to packing.

(VI) For heavy nuclei beyond $A = 180$ packing fraction(f) is positive. Thus we conclude that nuclei of the middle range are stable while light (below $A = 20$) and heavy (beyond $A = 200$) are unstable.

All nuclei have a tendency to move from a region of higher ~~of~~ packing fraction to lower packing fraction to attain higher stability. This is why fusion of light nuclei and fission of heavy nuclei takes place with a release of energy.

Binding energy:-

The mass of ~~the~~ any permanently stable nucleus is found to be less than ~~sum~~ the sum of the masses of the neutrons and protons which it contains. The fact is accounted for by the conversion of a part of the mass energy of the particles into energy binding; the relation between the change in mass and binding energy being given by Einstein's equation.

$$\text{Binding energy} = E_B = \Delta m c^2$$

The total binding energy for all atoms except the lighter atoms is given approximately by the emperical relation.

$$E_B = 15.6 Z \text{ MeV}$$

The binding energy of the nucleus with charge number Z and mass number A is given by

$$E_B = c^2 [Z m_p + (A-Z) m_n - M_N]$$

where M_N is the mass of the nucleus, m_p is the mass of the proton and m_n is the mass of the neutron.

The nuclear mass is therefore given by

$$M_N = Z m_p + (A-Z) m_n - E_B/c^2$$

and the atomic mass is

$$M = M_N + Z m_e = Z m_e + Z m_p + (A-Z) m_n - E_B/c^2$$

$$= Z M_H + (A-Z) m_n - E_B/c^2$$

where ~~Bm~~ m_e is the mass of the electron and M_H the mass of the hydrogen atom. The term E_b/c^2 represents the mass of equivalent of the total binding energy. ie the energy which must be added to the nucleus in order to break it ~~up~~ up into ~~Z~~ Z proton and (A−Z) neutrons.

~~Binding energy of the nucleus is less than the sum of the masses of the total number of proton and neutron in it.~~

Binding energy of a nucleus is therefore defined as the energy which must be supplied to the nucleus to break it into its constituent nucleons.

Since the mass of a nucleus is less than the sum of the masses of the total number of proton and neutrons in it, the binding energy is a positive quantity. The experimental value of nuclear binding energy varies from 2.23 MeV for deutron, the lightest stable atom containing more than one nucleon to 1640 MeV for the heaviest stable nucleus $^{209}_{83}Bi$.

Difference between mass defect and packing fraction:-

packing fraction $f = \frac{M-A}{A}$

where M is the actual mass (or weight) of the nucleus and A the mass number. But $M-A = \Delta m$ = mass defect.

$$\therefore f = \frac{\Delta m}{A} = \frac{\text{mass defect}}{\text{mass number (or number of nucleons)}}$$

Hence packing fraction is equal to mass defect per nucleon

Binding energy of the nucleus $= \Delta m c^2$ and binding energy per nucleon $= \frac{\Delta m}{A} c^2 = f c^2$

$\therefore$ Binding energy per nucleon $= c^2 \times$ packing fraction.

Binding energy per nucleon or Average binding energy is defined as binding energy per nucleon. It is the energy required to release a nucleon from the nucleus.