Exact Differential Eqn:

Ex. Solve

$$(1+4xy+2y^2)dx + (1+4xy+2x^2)dy = 0$$

Soln. Here, $M = 1+4xy+2y^2$,

$N = 1+4xy+2x^2$.

Now,

$$\frac{\partial M}{\partial y} = \frac{\partial}{\partial y}(1+4xy+2y^2) = 4x+4y$$

&

$$\frac{\partial N}{\partial x} = \frac{\partial}{\partial x}(1+4xy+2x^2) = 4y+4x.$$

Thus,

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

i.e. the condition of exactness is satisfied.

$\therefore$ The given diff. eqn. is exact.

$\therefore$ General soln. is

$$\int_{y\text{-const}} M\,dx + \int (\text{the term free from } x \text{ in } N)\,dy = c$$

$$\Rightarrow \int_{y\text{-const.}} (1+4xy+2y^2)\,dx + \int 1\,dy = c$$

$$\Rightarrow x + 2x^2y + 2xy^2 + y = c \quad \leftarrow \text{Ans.}$$

Ex. Solve: $(1+e^{x/y})\,dx + e^{x/y}(1-x/y)\,dy = 0$

Soln. Here, $M = 1+e^{x/y}$

$N = e^{x/y}\left(1-\frac{x}{y}\right)$.

Now,

$$\frac{\partial M}{\partial y} = \frac{\partial}{\partial y}\left\{1 + e^{x/y}\right\} = e^{x/y}\left(-\frac{x}{y^2}\right)$$

&

$$\frac{\partial N}{\partial x} = \frac{\partial}{\partial x}\left\{e^{x/y}(1 - x/y)\right\}$$

$$= e^{x/y}\,\frac{1}{y}(1 - x/y) + e^{x/y}\left(-\frac{1}{y}\right)$$

$$= \cancel{e^{x/y}\left(\frac{1}{y}\right)} - e^{x/y}\left(-\frac{x}{y^2}\right) - \cancel{e^{x/y}\left(\frac{1}{y}\right)}$$

$$= e^{x/y}\left(-\frac{x}{y^2}\right)$$

Thus,

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$ ∴ The cond^n of exactness is satisfied.

∴ The given eqn. is exact.

∴ The general soln. is

$$\int_{y\text{-const.}} (1 + e^{x/y})\,dx + \int (0)\,dy = c,$$ ∵ there is no term free from x in N.

$$\Rightarrow x + e^{x/y}\cdot y = c \longleftarrow \text{Ans.}$$

H.W. Solve

(1) $(x^4 + 4x^3y + 3x)\,dx + (x^4 + 4xy^3 + y + 1)\,dy = 0$

(2) $(x - 2e^x)\,dy + (y + x\sin x)\,dx = 0$

(3) $x\,dx - y\,dy = \dfrac{a^2(x\,dy - y\,dx)}{x^2 + y^2}$

(4) $x\,dx + y\,dy = \dfrac{y\,dx - x\,dy}{x^2 + y^2}$

(5) $(\cos x\tan y - \sin x\sec y)\,dx + (\sin x\sec^2 y + \cos x\tan^2 y\,\text{cosec}\,y)\,dy = 0$