Interpolation with unequal Intervals *

Lagrange's Interpolation formula for unequal Intervals:→

Let $f(x_0), f(x_1), f(x_2), \ldots, f(x_n)$ be the $(n+1)$ values of the function $y = f(x)$ corresponding to the values $x_0, x_1, x_2, \ldots, x_n$ of x; where $x_1 - x_0, x_2 - x_1, \ldots, x_n - x_{n-1}$ are not necessarily equal.

Let us define a polynomial $P_n(x)$ of degree n by

$$
\begin{aligned}
P_n(x) = {} & A_0(x-x_1)(x-x_2)\cdots(x-x_n) \\
& + A_1(x-x_0)(x-x_2)(x-x_3)\cdots(x-x_n) \\
& + A_2(x-x_0)(x-x_1)(x-x_3)(x-x_4)\cdots(x-x_n) \\
& + \cdots \\
& + A_n(x-x_0)(x-x_1)(x-x_2)\cdots(x-x_{n-1})
\end{aligned}
\quad \longrightarrow (1)
$$

where $A_0, A_1, A_2, \ldots, A_n$ are constants to be determined such that

$$P_n(x_0) = f(x_0),\ P_n(x_1) = f(x_1), \ldots, P_n(x_n) = f(x_n)$$

Putting $x = x_0, x_1, x_2, \ldots, x_n$ successively in (1), we get

$$P_n(x_0) = A_0(x_0-x_1)(x_0-x_2)\cdots(x_0-x_n)$$

$$\Rightarrow f(x_0) = A_0(x_0-x_1)(x_0-x_2)\cdots(x_0-x_n)$$

$$\therefore A_0 = \frac{f(x_0)}{(x_0-x_1)(x_0-x_2)\cdots(x_0-x_n)}$$

$$P_n(x_1) = A_1(x_1-x_0)(x_1-x_2)\cdots(x_1-x_n)$$

$$\Rightarrow A_1 = \frac{f(x_1)}{(x_1-x_0)(x_1-x_2)\cdots(x_1-x_n)}, \because P_n(x_1)=f(x_1)$$

Similarly,

$$A_2 = \frac{f(x_2)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)\cdots(x_2-x_n)}$$

$$\cdots\cdots\cdots\cdots$$

$$A_n = \frac{f(x_n)}{(x_n-x_0)(x_n-x_1)\cdots(x_n-x_{n-1})}$$

Now, putting these values of $A_0, A_1, \cdots, A_n$ in (1) we get

$$P_n(x) \equiv f(x) = \frac{(x-x_1)(x-x_2)\cdots(x-x_n)}{(x_0-x_1)(x_0-x_2)\cdots(x_0-x_n)} f(x_0)$$

$$+ \frac{(x-x_0)(x-x_2)\cdots(x-x_n)}{(x_1-x_0)(x_1-x_2)\cdots(x_1-x_n)} f(x_1)$$

$$+ \cdots\cdots$$

$$+ \frac{(x-x_0)(x-x_1)(x-x_2)\cdots(x-x_{n-1})}{(x_n-x_0)(x_n-x_1)(x_n-x_2)\cdots(x_n-x_{n-1})} f(x_n)$$

which is the required Lagrange's interpolation formula for unequal interpolation. #