

Ex. A third degree polynomial  $P(x)$  passes through the points  $(0, -1), (1, 1), (2, 1), (3, -2)$ . Find its value at  $x = 1.2$

Sol<sup>n</sup> First, we construct the following table.

$x$	$y$	$\Delta y$	$\Delta^2 y$	$\Delta^3 y$
0	-1			
1	1	2		
2	1	0	-2	
3	-2	-3	-3	-1

Here  $u = \frac{x-x_0}{h} = \frac{x-0}{1} = x$

By Newton's forward interpolation formula

$$\begin{aligned}
 P(x) = y &= y_0 + u \Delta y_0 + \frac{u(u-1)}{2} \Delta^2 y_0 + \frac{u(u-1)(u-2)}{6} \Delta^3 y_0 \\
 &= -1 + x \cdot 2 + \frac{1}{2} x(x-1)(-2) + \frac{1}{6} \{x(x-1)(x-2) \cdot (-1)\} \\
 &= -1 + 2x - x^2 + x - \frac{1}{6} (x^3 - 3x^2 + 2x) \\
 &= -1 + 2x - x^2 + x - \frac{1}{6} x^3 + \frac{1}{2} x^2 - \frac{1}{3} x \\
 &= -\frac{1}{6} x^3 - \frac{1}{2} x^2 + \frac{8}{3} x - 1
 \end{aligned}$$

$$\begin{aligned}
 \therefore P(1.2) &= -\frac{1}{6} (1.2)^3 - \frac{1}{2} (1.2)^2 + \frac{8}{3} (1.2) - 1 \\
 &= -0.288 - 0.72 + 3.2 - 1 \\
 &= 3.2 - 2.008 \\
 &= 1.192 \leftarrow \text{Ans}
 \end{aligned}$$

144	
12	
1758	
1.728	0.288
6	
3.200	0.288
2.008	0.720
1.192	1.000
	2.008

Ex. A second degree polynomial passes through  $(0,1)$ ,  $(1,3)$ ,  $(2,7)$  and  $(3,13)$ . Find the poly.

Sol. We construct the following difference table

$x$	$y$	$\Delta y$	$\Delta^2 y$	$\Delta^3 y$
0	1			
1	3	2		
2	7	4	2	0
3	13	6	2	

Here,  $u = \frac{x-x_0}{h} = \frac{x-0}{1} = x$ .

Hence, by Newton's forward interpolation formula

$$f(x) = y_0 + u \Delta y_0 + u(u-1) \frac{\Delta^2 y_0}{2}$$

$$= 1 + x \cdot 2 + x(x-1) \cdot \frac{2}{2}$$

$$= 1 + 2x + x^2 - x$$

$$= x^2 + x + 1 \quad \leftarrow \text{Ans.}$$

Ex. The rainfall of a place was recorded as

Day (August)	16	18	20	22
Rainfall (in mm.)	2.7	1.9	0.7	2.9

What was the rainfall on 17<sup>th</sup> August?

