Putting $e_x = \alpha i$, $e_y = \beta j$, $e_z = \gamma k$

$$du = dx e_1 + dy e_2 + dz e_3$$

These e_x, e_y & e_z are called base vectors having the metric information, so

$$ds^2 = dx^2 e_x e_x + dy^2 e_y e_y + dz^2 e_z e_z$$

Then
$$\underline{G} = \begin{vmatrix} e_x e_x & 0 & 0 \\ 0 & e_y e_y & 0 \\ 0 & 0 & e_z e_z \end{vmatrix}$$

This means that

$e_x e_y = e_y e_z = e_z e_x = 0$ means that

e_x, e_y & e_z are mutually orthogonal.

Here
$$\underline{G} = \begin{vmatrix} e_x e_x & e_x e_y & e_x e_z \\ e_y e_x & e_y e_y & e_y e_z \\ e_z e_x & e_z e_y & e_z e_z \end{vmatrix}$$

Replacing x, y and z by 1, 2, & 3

$$e_{11} = g_{11}, \quad e_{12} = g_{12} \; - \; - \; -$$

$$\therefore \underline{G} = \begin{vmatrix} g_{11} & g_{12} & g_{13} \\ g_{21} & g_{22} & g_{23} \\ g_{31} & g_{32} & g_{33} \end{vmatrix}$$

For material information, in general co-ordinate system another important function of the matric tensor is to relate the covariant and contravariant components of a base vector within the given co-ordinate system.

Suppose.

g_{jk} = pairwise = g_{kj} – Symmetric tensor

$g_{jk} = -g_{kj}$ – anti symmetric tensor.

Co-ordinate system, Base vectors, Covariant & Contravariant

Let ~~two~~ a nonorthogonal co-ordinate system (eg- crystallographic) having ~~base~~ base vectors with distinct magnitudes usualy bent or curved,

$A = a_x i + a_y j + a_z k$

a_x, a_y & a_z are the component of A along i, j & k that seperated xy (12) xz (13) & yz (23) plane

Considering a point P away from origin in space with a vector v having local basis, we may specify our local basis at P in one of two ways.

① setting local axes at P using local co-ordinate curve which parallel to Cartesian Co-ordinate axes. Then ~~choose~~ choose a set tangent to each of the local axes at P and call them as e_1, e_2 & e_3 and V. specify as a a linear Combination of these three vectors. Known as contravariant components of the vectors.

② Setting another three sets of base vectors which are e_1^*, e_2^* & e_3^*; mutually perpendicular to each other. The linear combination these three ~~const~~ base vectors ~~const~~ constitute the Vector V. to the local cordinate systems; known as Covariant components of the vectors. omitting 'i' j & k.

~~Since Euclidean geomatry that two non-parallel with the combination points~~

Since Euclidean geomatry two non parallel intersecting line constitute a plane and each product with co-ordinate pair.

Space collections of points with co-ordinates pair. The n-dimensional & m-dimensional space may be used to determine a new unique (m+n) dimensional product space. The product space it may be result of lines or curves: i.e →

① Contravariant base vectors related to the curve, usually denoted by superscripts i.e

$\{e^1, e^2\ e^3\} \rightarrow$ for K^* System

$\{e^{1*}\ e^{2*}\ e^{3*}\}$ — for K^* System

② Covariant base vectors related to the lines usually denoted by subscripts

$\{e_1, e_2, e_3\} \longrightarrow$ for K system

$(e_1^*, e_2^*, e_3^*) \longrightarrow$ for K^* "

① The vector v in its contravariant and covariant components is

$$v = v^1 e^1 + v^2 e^2 + v^3 e^3 = v_1 e_1 + v_2 e_2 + v_3 e_3$$

$$v^* = v^{1*} e^{1*} + v^{2*} e^{2*} + v^{3*} e^{3*} = v_1^* e_1^* + v_2^* e_2^* + v_3^* e_3^*$$

② For dyad G.

ⓐ Covariant $g_{jk} = e^j \cdot e^k$ | j & k are 1, 2, 3 - -

ⓑ Contravariant $g^{jk} = e_j \cdot e_k$

---

x-axis — 1 axis — plane YZ → 23 plane

y " 2 " " xz — 13

z " 3 " " xy — 12

So, $e_x = e_1$ $e_y = e_2$ $e_z = e_3$

e

Lect-III

Kronecker Delta and Identity Matrix

All these information about contravariant and covariant base of basis vectors may be summarised in a single equation. Let we must first introduce a particular symbol named as Kronecker's Delta [Leopold Kronecker, a German mathematician) ie $\delta^j_k \rightarrow$ appears to mix covariant and contravariant indices.

$$\delta^j_k = 1 \quad \text{when} \quad j = k$$
$$\delta^j_k = 0 \quad \text{when} \quad j \neq k$$

Now summarise the relationship betn contravariant and covariant base vectors as

$$e^j \cdot e_k = \delta^j_k = e_k \cdot e^j = \delta^k_j$$

Thus $\delta^1_1 = 1$ $\quad \delta^2_1 = 0$ $\quad \delta^3_1 = 0$

$\delta^2_1 = 0$ $\quad \delta^2_2 = 1$ $\quad \delta^3_2 = 0$

$\delta^3_1 = 0$ $\quad \delta^3_2 = 0$ $\quad \delta^3_3 = 0$

Thus square matrix $\underline{I}$

$$\underline{I} = \begin{vmatrix} \delta^1_1 & \delta^2_1 & \delta^3_1 \\ \delta^2_1 & \delta^2_2 & \delta^2_3 \\ \delta^3_1 & \delta^3_2 & \delta^3_3 \end{vmatrix}$$

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For any n-ads $\underline{X}$

$$\underline{I} \cdot \underline{X} = \underline{X} \cdot \underline{I} = \underline{X}$$

Dyad components.

(Covariant, contravariant, Mixed dyad)

(1) For typical dyads

$$\underline{D} = A B$$

The vectors A & B may have individually

(a) Covariant — Covariant

(b) Contravariant — Contravariant

(c) Covariant — Contravariant

(d) Contravariant — Covariant

ie

(1) Covariant $a_j b_k = c_{jk}$

(2) Mixed — $a_j b^k = c_j^{\ k}$

(3) Mixed $a^j b_k = c_k^{\ j}$

(4) Contravariant. $a^j b^k = c^{jk}$

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