Newton-Gregory Backward Interpolation formula :-

Let $f(a), f(a+2h), \ldots, f(a+nh)$ be the $(n+1)$ values of the function $y = f(x)$ corresponding to the equally spaced values $a, a+h, \ldots, a+nh$ of x.

Now we shall find an approximate representation of $f(x)$ in the form of a polynomial given by

$$P_n(x) = A_0 + A_1(x-a-nh) + A_2(x-a-nh)(x-a-nh+h)$$
$$+ A_3(x-a-nh)(x-a-nh+h)(x-a-nh+2h)$$
$$+ \cdots$$
$$+ A_n(x-a-nh)(x-a-nh-h)(x-a-nh+2h)$$
$$\cdots (x-a-h) \longrightarrow (2)$$

where $A_0, A_1, A_2, \ldots, A_n$ are constants to be determined s.t.

$$P_n(a+nh) = f(a+nh)$$
$$P_n(a+nh-h) = f(a+n-h)$$
$$\cdots$$
$$P_n(a) = f(a)$$

Putting $x = a+nh, a+nh-h, \ldots$ successively, we get

$$P_n(a+nh) = A_0$$
$$\Rightarrow f(a+nh) = A_0$$

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Also,

$$P_n(a+nh-h) = A_0 + A_1(-h)$$

$$\Rightarrow f(a+nh-h) = f(a+nh) \;\; A_1 h$$

$$\Rightarrow A_1 h = f(a+nh) - f(a+nh-h)$$

$$= \nabla f(a+nh)$$

$$\Rightarrow A_1 = \frac{\nabla f(a+nh)}{h}$$

Next,

$$P_n(a+nh-2h) = A_0 + A_1(-2h) + A_2(-2h)(-h)$$

$$\Rightarrow f(a+nh-2h) = f(a+nh) - 2h \cdot \frac{f(a+nh) - f(a+nh-h)}{h}$$

$$+ A_2 h \lfloor 2$$

$$\Rightarrow A_2 h^2 \lfloor 2 = f(a+nh-2h) - 2f(a+nh-h) + f(a+nh)$$

$$= \nabla^2 f(a+nh)$$

$$\therefore A_2 = \frac{\nabla^2 f(a+nh)}{\lfloor 2 \, h^2}$$

Similarly,

$$A_3 = \frac{\nabla^3 f(a+nh)}{\lfloor 3 \, h^3}, \; \cdots$$

$$\cdots A_n = \frac{\nabla^n f(a+nh)}{\lfloor n \, h^n}$$

Putting these values of $A_0, A_1, A_2, \cdots, A_n$ in the eqn. (1), we get

$$f(x) \simeq P_n(x) = f(a+nh) + \frac{\nabla f(a+nh)}{h}(x-a-nh)$$

$$+ \frac{\nabla^2 f(a+nh)}{\underline{|2}\, h^2}(x-a-nh)(x-a-nh+h)$$

$$+ \frac{\nabla^3 f(a+nh)}{\underline{|3}\, h^3}(x-a-nh)(x-a-nh+h)(x-a-nh+2h)$$

$$+ \cdots\cdots\cdots$$

$$+ \frac{\nabla^n f(a+nh)}{\underline{|n}\, h^n}(x-a-nh)(x-a-nh+h)$$

$$\cdot(x-a-nh+2h)\cdots(x-a-h) \longrightarrow (2)$$

Let $u = \dfrac{x-(a+nh)}{h} \Rightarrow x = (a+nh) + hu$

$$\therefore (2) \Rightarrow f(x) = f(a+nh) + \frac{\nabla f(a+nh)}{h} hu$$

$$+ \frac{\nabla^2 f(a+nh)}{\underline{|2}\, h^2} hu(hu+h)$$

$$+ \cdots\cdots\cdots$$

$$+ \frac{\nabla^n f(a+nh)}{\underline{|n}\, h^n} hu(hu+h)(hu+2h)\cdots(hu-\overline{n-1}h)$$

or,

$$f(x) = f(a+nh) + u\nabla f(a+nh) + \frac{u(u+1)}{\underline{|2}}\nabla^2 f(a+nh)$$

$$+ \frac{u(u+1)(u+2)}{\underline{|3}}\nabla^3 f(a+nh) + \cdots$$

$$+ \frac{u(u+1)(u+2)\cdots(u+n-1)}{\underline{|n}}\nabla^n f(a+nh)$$

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which is Newton's backward int. formula.