Exercise 10.2

Q. No. 11.

দেখুওৱা যে $2\hat{i}-3\hat{j}+4\hat{k}$ আৰু $-4\hat{i}+6j-8\hat{k}$ ভেক্টৰ দুটা একৰেখীয়।

(Show that the vector $2i-3j+4k$ and $-4i+6j-8k$ are co-llinear)

Solⁿ. Let $\vec{a} = 2i-3\hat{j}+4k \Rightarrow a_1 = 2,\ a_2 = -3,\ a_3 = 4$

$\vec{b} = -4i+6j-8k \Rightarrow b_1 = -4,\ b_2 = 6,\ b_3 = -8.$

ইয়াত (here),

$$\frac{a_1}{b_1} = \frac{a_2}{b_2} = \frac{a_3}{b_3} \Rightarrow \frac{2}{-4} = \frac{-3}{6} = \frac{4}{-8} = -\frac{1}{2}, \text{ সঁচা, true}$$

∴ Given vectors are co-llinear.

Q. No. 10. $5\hat{i}-\hat{j}+2\hat{k}$ ভেক্টৰৰ দিশত হোৱা এটা ভেক্টৰ উলিওৱা যাৰ মান 8 একক। [Find a vector in the direction of $5\hat{i}-\hat{j}+2k$ whose magnitude is 8 unit.]

Solⁿ $5\hat{i}-j+2\hat{k}$ ৰ দিশত একক ভেক্টৰ (unit vector along $5i-j+2k$)

$$= \frac{5\hat{i}-j+2k}{|5i-j+2k|} = \frac{5i-j+2k}{\sqrt{5^2+1^2+2^2}} = \frac{5i-j+2k}{\sqrt{30}}$$

∴ নির্ণেয় ভেক্টৰ (required vector)

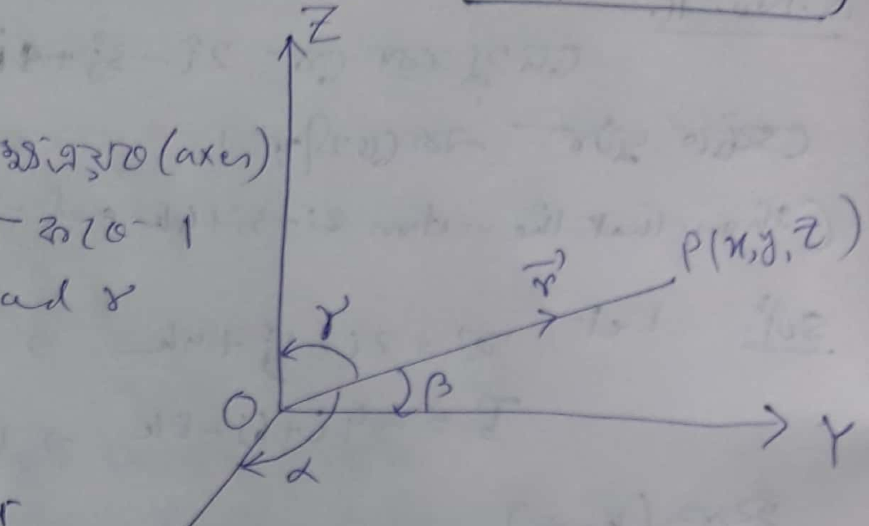
$$= 8 \times \frac{5i-j+2k}{\sqrt{30}}$$

$$= \frac{40}{\sqrt{30}}i - \frac{8}{\sqrt{30}}j + \frac{16}{\sqrt{30}}k \leftarrow \text{Ans.}$$

দিশাংক (Direction cosine), দিশানুপাত (Direction Ratio)

$\vec{OP} = xi + yj + zk$

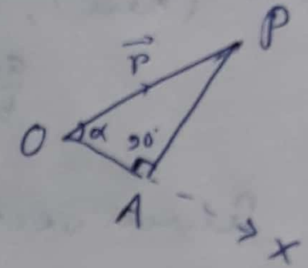
ধৰোঁ (let), $\vec{OP}$ ভেক্টৰটোৱে অক্ষত্ৰয়ৰ (axes) লগত α, β আৰু γ কোণ কৰে।
(Let $\vec{OP}$ makes the α, β, and γ with the axes).



তেতিয়া, (then)

α, β, γ ক $\vec{OP}$-ৰ দিশ কোণ (direction angle) বোলে

$\cos\alpha, \cos\beta, \cos\gamma$-ক $\vec{OP}$-ৰ দিশাংক (direction cosine) বোলে।



$\triangle OAP \Rightarrow \cos\alpha = \frac{x}{r}$, $\left(r = |\vec{r}| \right)$, $r = \sqrt{x^2+y^2+z^2}$

Similarly,
$\cos\beta = \frac{y}{r}$, $\cos\gamma = \frac{z}{r}$.

we write, $l = \cos\alpha$, $m = \cos\beta$, $n = \cos\gamma$.

$\therefore P(x,y,z) \equiv P(lr, mr, nr)$.

দিশানুপাত (direction ratio): lr, mr, nr - ক $\vec{r}$ ভেক্টৰৰ দিশানুপাত বোলে। In general we write,

$lr = a$, $mr = b$, $nr = c$. $\Rightarrow \boxed{l = \frac{a}{r}, \; m = \frac{b}{r}, \; n = \frac{c}{r}}$

Also, we can say, x, y, z are direction ratio of $\vec{OP}$.

Note: $\boxed{l^2 + m^2 + n^2 = 1}$, but in general $\boxed{a^2+b^2+c^2 \neq 1}$
(we shall prove this result later on.)

Q. No. (12).

Exercise 10.2

(18)

$\hat{i} + 2\hat{j} + 3\hat{k}$ ভেক্টৰটোৰ দিশাংক উলিওৱা।

[Find the direction cosine (d.c.) of the vector $\hat{i}+2\hat{j}+3\hat{k}$]

Soln. Let $\vec{r} = \hat{i}+2\hat{j}+3\hat{k}$

∴ দিশানুপাত (direction ratios) a, b, c হ'ল

$$a = 1,\ b = 2,\ c = 3$$

যদি দিশাংক (d.c.) l, m, n হয়, তেন্তে (then)

$$l = \frac{a}{|\vec{r}|} = \frac{1}{\sqrt{1^2+2^2+3^2}} = \frac{1}{\sqrt{14}}$$

$$m = \frac{b}{|\vec{r}|} = \frac{2}{\sqrt{14}}$$

$$n = \frac{c}{|\vec{r}|} = \frac{3}{\sqrt{14}}$$

∴ দিশাংক (d.c.) হ'ল $\left(\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}\right)$ ← Ans

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13. A(1, 2, −3) আৰু B(−1, −2, 1) বিন্দু সংযোগী A ৰ পৰা B লৈ দিশযুক্ত ভেক্টৰটোৰ দিশাংক উলিওৱা। [find the d.c. of the vector $\vec{AB}$]

Soln ইয়াত (here), $\vec{AB} = (-1-1)\hat{i} + (-2-2)\hat{j} + \{1-(-3)\}\hat{k}$

$$\Rightarrow \vec{r} = \vec{AB} = -2\hat{i} - 4\hat{j} + 4\hat{k}.$$

Also, here, $a = -2,\ b = -4,\ c = 4.$

$$\therefore\ l = \frac{a}{|\vec{r}|} = \frac{-2}{\sqrt{(-2)^2+(-4)^2+4^2}} = \frac{-2}{\sqrt{36}} = \frac{-2}{6} = \frac{-1}{3}$$

$$m = \frac{b}{|\vec{r}|} = \frac{-4}{6} = -\frac{2}{3}$$

$$n = \frac{c}{|\vec{r}|} = \frac{4}{6} = \frac{2}{3}$$

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