

Homogeneous & Homothetic Function

A homothetic function is a monotonic transformation of a function which is homogeneous. The term monotonic transformation (or monotone transformation) refers to a transformation by a strictly increasing function. That is, 'monotonic transformation' is more accurately called a "positive monotonic transformation". In consumer theory, a consumer's preferences are called homothetic, if they can be represented by a utility function which is homogeneous of degree one. For example, in an economy, with two goods, x, y , homothetic preferences can be represented by a utility function, 'u' that has following property —

$$u(ax, ay) = au(x, y)$$

In mathematics, a homothetic function is a monotonic transformation of a

function which is homogeneous

Example:

For the function $f(x_1, x_2) = Ax_1^a x_2^b$,
test the homogeneity of function.

Solution:

$$f(tx_1, tx_2) = A(tx_1)^a (tx_2)^b$$

$$= At^{a+b} x_1^a x_2^b$$

$$= t^{a+b} Ax_1^a x_2^b$$

$$= t^{a+b} f(x_1, x_2)$$

So, f is a homogeneous of degree $a+b$.

Example:

Given the function, check whether the function is homogeneous function or not.

$$f(x, y, z) = x^5 y^2 z^3$$

Solution:

The given function -

$$f(x, y, z) = x^5 y^2 z^3$$

Multiplying the above function by some factor 'd', we get -

$$f(dx, dy, dz) = (dx)^5 (dy)^2 (dz)^3$$

$$= d^{10} x^5 y^2 z^3$$

$$= d^{10} f(x, y, z)$$

Hence, the function is homogeneous of degree 10.

Example:

Consider the production function -

$$y = f(x_1, x_2) = x_1^{1/4} x_2^{1/3}$$

Determine whether the production function is homogeneous. If so, of what degree.

Solution:

Given the function -

$$y = f(x_1, x_2) = x_1^{1/4} x_2^{1/3}$$

Multiplying the function by some factor ' β ', we get -

$$\begin{aligned} f(\beta x_1, \beta x_2) &= (\beta x_1)^{1/4} (\beta x_2)^{1/3} \\ &= \beta^{1/4 + 1/3} x_1^{1/4} x_2^{1/3} \\ &= \beta^{7/12} x_1^{1/4} x_2^{1/3} \\ &= \beta^{7/12} f(x_1, x_2) \end{aligned}$$

Hence the function is a homogeneous function. And it is homogeneous of degree $7/12$.