

(7)

Transpose of a Matrix: - When the rows and columns of a Matrix A are interchanged - so that its first row becomes the first column and vice-versa - we obtain the transpose of A , which is denoted by A' . For example,

$$(i) A = \begin{bmatrix} 3 & 8 & -9 \\ 1 & 0 & 4 \end{bmatrix}_{2 \times 3} \quad \therefore A' = \begin{bmatrix} 3 & 1 \\ 8 & 0 \\ -9 & 4 \end{bmatrix}_{3 \times 2}$$

$$(ii) B = \begin{bmatrix} 3 & -4 \\ -1 & 7 \end{bmatrix}_{2 \times 2} \quad \therefore B' = \begin{bmatrix} 3 & -1 \\ -4 & 7 \end{bmatrix}_{2 \times 2} \quad (ii)$$

By definition, if a matrix A is $m \times n$, then its transpose A' must be $n \times m$. An $n \times n$ square matrix possesses a transpose with the same dimension.

(8) Square Matrix: - A matrix having the same number of rows as it has columns is called a square matrix. For example,

$$(i) A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}_{2 \times 2} \quad (ii) B = \begin{bmatrix} 1 & 0 & 4 \\ 0 & 3 & 7 \\ 4 & 6 & 2 \end{bmatrix}_{3 \times 3}$$

(9) Diagonal Matrix: - A square matrix having all the elements zero except the principal diagonal elements, is called a diagonal matrix. For example,

$$(i) A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}_{2 \times 2} \quad B = \begin{bmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 3 \end{bmatrix}_{3 \times 3}$$

(10) Identity Matrix (or unit Matrix): A square matrix in which each diagonal elements are one, all other elements being zero is called a unit or identity matrix, it is denoted by I . For example,

(i) $I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}_{2 \times 2}$ (ii) $I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}_{3 \times 3}$

(11) Scalar Matrix: A square matrix whose all elements except those in the main diagonal are zero and the diagonal elements are all equal is called a scalar matrix. For example,

(i) $A = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}_{2 \times 2}$ (ii) $B = \begin{bmatrix} 10 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 10 \end{bmatrix}_{3 \times 3}$

(12) Symmetric Matrix: A square matrix $A = (a_{ij})$ is said to be symmetric if $a_{ij} = a_{ji}$ for all i and j on both sides of the diagonal

ie if $a_{12} = a_{21}$, $a_{13} = a_{31}$, $a_{23} = a_{32}$ etc.

For example, (i) $A = \begin{bmatrix} a & g \\ g & a \end{bmatrix}_{2 \times 2}$ (ii) $B = \begin{bmatrix} 2 & 6 & -3 \\ 6 & 3 & 7 \\ -3 & 7 & 4 \end{bmatrix}_{3 \times 3}$

$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ (iii) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

(13) Skew Symmetric Matrix: A square matrix A

$A = (a_{ij})$ is said to be skew symmetric

if $a_{ij} = -a_{ji}$ for all i and j on both

sides of the diagonal i.e. $a_{12} = -a_{21}$, $a_{13} = -a_{31}$,

$a_{23} = -a_{32}$ etc. and $a_{11} = a_{22} = a_{33} \dots = a_{nn} = 0$

For example:-

$$(i) \begin{bmatrix} 0 & -2 & -4 \\ 2 & 0 & -5 \\ 4 & 5 & 0 \end{bmatrix}_{3 \times 3}$$

$$(ii) \begin{bmatrix} 0_{11} & a_{12} & a_{13} \\ -a_{21} & 0_{22} & a_{23} \\ -a_{31} & -a_{32} & a_{33} \end{bmatrix}_{3 \times 3}$$

(14) Idempotent Matrix: A symmetric matrix

is called Idempotent matrix when it ~~is~~ produces itself as a result of multiplied by itself. For example;

$$(i) A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}_{2 \times 2} \text{ then } A^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$(ii) B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \text{ then } B \cdot B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\ = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = B$$

$$\therefore B^2 = B$$