Dyad as Matrix :-

Let $A = ai + bj + ck$

&

$D = di + ej + fk$

$\therefore AD = adii + aeij + afik + bdji + bejj$

$+ bfjk + cdki + cekj + cfkk$

Putting

$a = a_1$ & $d = d_1$

$b = a_2$ $e = d_2$

$c = a_3$ $f = d_3$

$\therefore AD = a_1d_1ii + a_1d_2ij + a_1d_3ik + a_2d_1ji +$

$a_2d_2jj + a_3d_3jk + a_3d_1ki + a_3d_2kj +$

a_3d_3kk

Setting

$a_1d_1 = \mu_{11}$ $a_2d_1 = \mu_{21}$

$a_1d_2 = \mu_{12}$ $a_2d_2 = \mu_{22}$

$a_1d_3 = \mu_{13}$ $a_2d_3 = \mu_{23}$

$a_3d_1 = \mu_{31}$

$a_3d_2 = \mu_{32}$

$a_3d_3 = \mu_{33}$

$\therefore$ Thus dyad AD as

$AD = \mu_{11}ii + \mu_{12}ij + \mu_{13}ik +$ ———

These may arrange as 3×3 square matrix

$$\begin{vmatrix} \mu_{11} & \mu_{12} & \mu_{13} \\ \mu_{21} & \mu_{22} & \mu_{23} \\ \mu_{31} & \mu_{32} & \mu_{33} \end{vmatrix}$$

The component of all dyads of a given dimension can be represented as a square matrix of $n\times n$ order which is not the commulative law ie

$$AD \neq DA$$

Co-ordinate Transformation

Let we have two co-ordinate system K & K^* labelled by (x, y, z) & $(x^* \; y^* \; z^*)$ for same two system

ie

$x = x(x^*, y^*, z^*)$ & $x^* = x^*(x, y, z)$

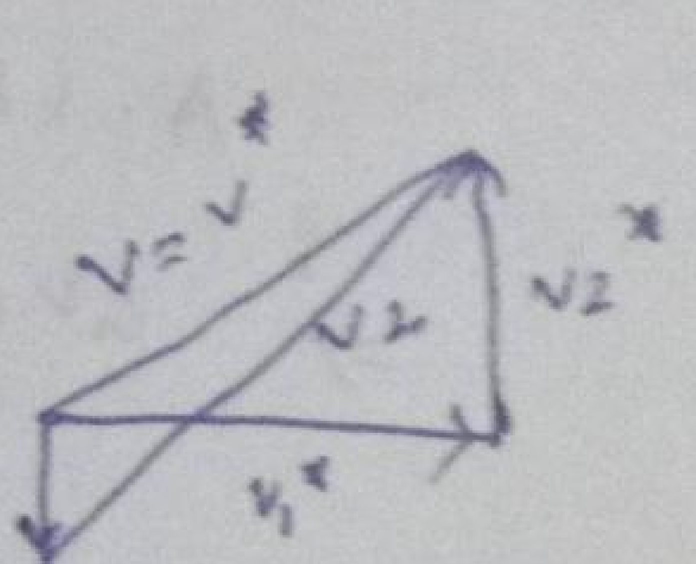
$y = y(x^* \; y^* \; z^*)$ & $y^* = y^*(x, y, z)$

$z = z(x^* \; y^* \; z^*)$ & $z^* = z^*(x, y, z)$

For simmplicity considering two vector V & V^* having identical value. ie

$$V = V^*$$

But their components in K & K^* system are not same.



For 2D case.

$$V = V_1 + V_2 \quad \& \quad V^* = V_1^* + V_2^*$$

(From the figure)

so, that

$$\left.\begin{aligned} v_1^* &= v_1^*(v_1, v_2) \\ v_2^* &= v_1(v_1, v_2) \end{aligned}\right\} \text{ — } k^* \text{ system}$$

&

$$\left.\begin{aligned} v_1 &= v_1^*(v_1^*, v_2) \\ v_2 &= v_2(v_1, v_2) \end{aligned}\right\} \text{ — } k \text{ system}$$

Besides $v \cdot v = v^* \cdot v^*$

$\Rightarrow \quad v^2 = v^{*2}$,

$\Rightarrow \quad \underline{v = v^*}$

Now for a ~~B~~ dyad $\underline{S}$ in both k & $\hat{k}$ system

$\underline{S} = AB$ & $\underline{S}^* = A^* B^*$

Wheather $\underline{S} = \underline{S}^*$. we have ~~to~~ ~~...~~ to ~~prove~~ Prove it -

Proof.

Let $AB = A^* B^*$

Pre multiply

$$A \cdot (AB) = A \cdot (A^* B^*)$$

$$\Rightarrow \quad a^2 B = (A \cdot A^*) B^*$$

$$\therefore \quad B = \left(\frac{A \cdot A^*}{a^2}\right) B^*$$

Putting $\beta = \left(\frac{A \cdot A^*}{a}\right)$

$\therefore \; B = \beta B^*$

This means that B is the function of B^*

Again,
$$AB \cdot B = A^* B^* \cdot B$$
$$A b^2 = A^* (B^* \cdot B)$$
$$A = A^* \frac{(B^* \cdot B)}{b^2}$$

Putting the value of $B^* = \frac{B}{\beta}$

$$\therefore \; A = A^* \frac{(B \cdot B)}{\beta b^2} = \frac{A^*}{\beta}$$

Hence $A \cdot B = \frac{A^*}{\beta} \cdot \beta B^* = A^* \cdot B^*$

$$\therefore \; \boxed{\underline{S} = \underline{S}^*}$$

Matrix or Fundamental Tensor:-

The quantity $ds^2 = dr \cdot dr$, means the square of magnitude of dr, but it may or may not represent a true length in meters or centimeter. Let α, β & γ be metric terms along x, y & z direction respectively i.e.

$$du = \alpha dx\, i + \beta dy\, j + \gamma dz\, k$$

$$\therefore \; ds^2 = du \cdot du$$

Putting $e_x = \alpha i$, $e_y = \beta j$, $e_z = \gamma k$

$\therefore$ $d\mathbf{r} = dx e_1 + dy e_2 + dz e_3$

These e_x, e_y & e_z are called base vectors having the metric information, so

$$ds^2 = dx^2 e_x e_x + dy^2 e_y e_y + dz^2 e_z e_z$$

Then $\underline{G} = \begin{vmatrix} e_x e_x & 0 & 0 \\ 0 & e_y e_y & 0 \\ 0 & 0 & e_z e_z \end{vmatrix}$

This means that

$e_x e_y = e_y e_z = e_z e_x = 0$ means that

e_x, e_y & e_z are mutually orthogonal.

Here $\underline{G} = \begin{vmatrix} e_x e_x & e_x e_y & e_x e_z \\ e_y e_x & e_y e_y & e_y e_z \\ e_z e_x & e_z e_y & e_z e_z \end{vmatrix}$

Replacing x, y and z by 1, 2, & 3

$e_{11} = g_{11}$, $e_{12} = g_{12}$ - -

$\therefore$ $\underline{G} = \begin{vmatrix} g_{11} & g_{12} & g_{13} \\ g_{21} & g_{22} & g_{23} \\ g_{31} & g_{32} & g_{33} \end{vmatrix}$