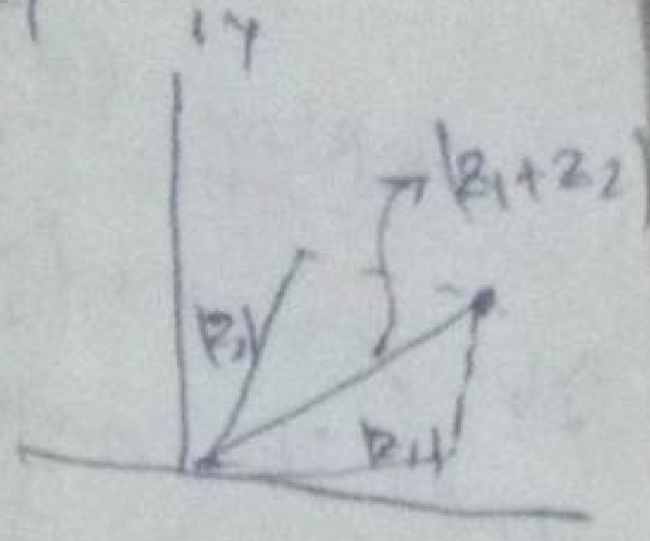
Hence $|z_1+z_2|^2 \le |z_1|^2 + 2|z_1||z_2| + |z_2|^2$

$= (|z_1| + |z_2|)^2$



$\therefore \quad |z_1 + z_2| \le |z_1| + |z_2|$

Proved

De-Moiver's Theorem :-

If $z_1 = x_1 + iy_1 = r_1(\cos\theta_1 + i\sin\theta_1)$

&

$z_2 = x_2 + iy_2 = r_2(\cos\theta_2 + i\sin\theta_2)$

Then $z_1z_2 = \{r_1(\cos\theta_1 + i\sin\theta_1)\}\{r_2(\cos\theta_2 + i\sin\theta_2)\}$

$= r_1r_2\{(\cos\theta_1\cos\theta_2 - \sin\theta_1\sin\theta_2) + i(\sin\theta_1\cos\theta_2 + \cos\theta_1\sin\theta_2)\}$

$= r_1r_2 \cos(\theta_1+\theta_2) + i\sin(\theta_1+\theta_2)$

&

The generalization of this eqn leads -

$z_1z_2 \cdots z_n = \{r_1r_2r_3 \cdots r_n\}\{\cos(\theta_1+\theta_2+\cdots\theta_n) + i\sin(\theta_1+\theta_2+\cdots+\theta_n)\}$

$\Rightarrow$ If $z_1 = z_2 = \cdots = z_n = z$, then

$z^n = \{\cancel{r(\cos\theta + i\sin\theta)}\}^n = \{r(\cos n\theta + i\sin n\theta)\}^1$

$z^n = r^n(\cos n\theta + i\sin n\theta)$

This is known as De-Moivre's Theorem.

$\frac{1}{3}$

Roots of complex numbers

If a number ω is called an nth root of a complex number z if $w^n = z$

$\Rightarrow \omega = z^{1/n}$

$$\therefore z^{1/n} = \{r(\cos\theta + i\sin\theta)\}^{\frac{1}{n}}$$

$$= r^{\frac{1}{n}}\left\{\cos\left(\frac{\theta_p + 2k\pi}{n}\right) + i\sin\left(\frac{\theta_p + 2k\pi}{n}\right)\right.$$

where $k = 0, 1, 2 \ldots (n-1)$

where $0 \le \theta_p < 2\pi$,

Problem find the square root of i

Soln Here $z = i$

$$\therefore (i)^{1/2} = r^{1/2}\left[\cos\left(\frac{\theta_p + 2\pi k}{2}\right) + i\sin\left(\frac{\theta_p + 2\pi k}{2}\right)\right]$$

The principal $\arg z = \theta_p = \pi/2$, because $x = 0$, $y = 1$, $r = 1$

For $k = 0$, we obtain

$$(i)^{1/2}_{k=0} = \cos\pi/4 + i\sin\pi/4$$

$$= \frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}}(1+i)$$

For $k = 1$,

$$(i)^{1/2}_{k=1} = \cos(\pi/4 + \pi) + i\sin(\pi/4 + \pi)$$

$$= \cos\pi/4 \cos\pi + i\sin\pi/4 \cdot \cos\pi$$

$\Delta x \to 0$

$\Delta y \to 0$

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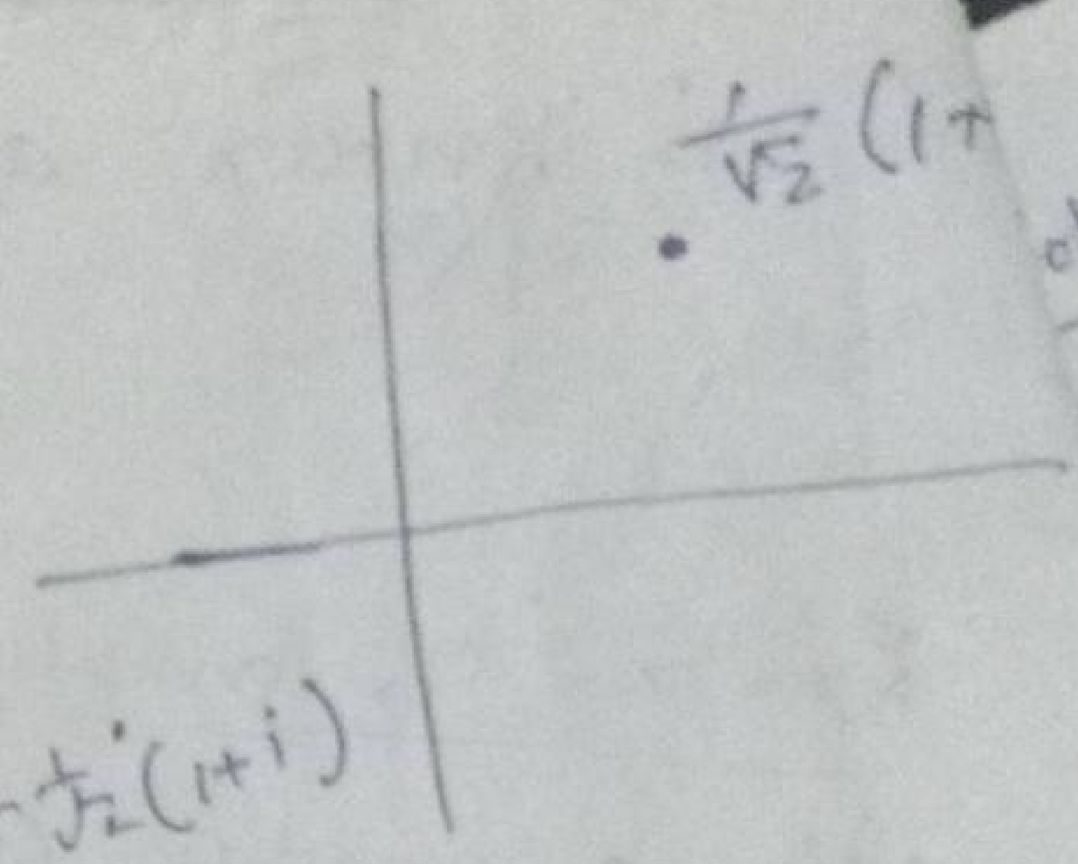
— i.e. The equation (2) reduces

Problem $= \frac{1}{\sqrt{2}}(1+i)$.

Thus results (i) $z_2 = \pm\frac{1}{\sqrt{2}}(1+i)$



Prob. Prove $|z_1+z_2| \le |z_1|+|z_2|$

Soln Analytically. Let $z_1 = x_1+iy_1$, $z_2 = x_2+iy_2$.

Then $\sqrt{(x_1+x_2)^2+(y_1+y_2)^2} \le \sqrt{x_1^2+y_1^2} + \sqrt{x_2^2+y_2^2}$

This will be true if

Squaring both sides, $(x_1+x_2)^2+(y_1+y_2)^2 \le x_1^2+y_1^2 + 2\sqrt{(x_1^2+y_1^2)(x_2^2+y_2^2)} + x_2^2+y_2^2$

$\therefore$ $x_1x_2+y_1y_2 \le \sqrt{(x_1^2+y_1^2)(x_2^2+y_2^2)}$

or squaring both sides. $x_1^2x_2^2 + 2x_1x_2y_1y_2 + y_1^2y_2^2 = x_1^2x_2^2 + x_1^2y_2^2 + y_1^2x_2^2 + y_1^2y_2^2$

$\Rightarrow$ $2x_1x_2y_1y_2 \le x_1^2y_2^2 + y_1^2x_2^2$

But this is equivalent to $(x_1y_2-x_2y_1)^2 \ge 0$, which is true. Reverse the steps, which reversible, proves the result.

S-4/L-3

Roots of complex number :- If a number w is called ... complex no. z if $w^n = z$

$\Rightarrow w = z^{\frac{1}{n}}$

$z^{\frac{1}{n}} = \left\{ r(\cos\theta + i\sin\theta) \right\}^{\frac{1}{n}}$

$= r^{\frac{1}{n}} \left\{ \cos\left(\frac{\theta_p + 2k\pi}{n}\right) + i\sin\left(\frac{\theta_p + 2k\pi}{n}\right) \right\}$; where $0 \le \theta_p \le 2\pi$

where $K = 0, 1, 2, \ldots (n-1)$

Problem :- Find the square root of i

Soln $z = i$.

$(i)^{\frac{1}{2}} = r^{\frac{1}{2}} \left\{ \cos\left(\frac{\theta_p + 2\pi k}{2}\right) + i\sin\left(\frac{\theta_p + 2\pi k}{2}\right) \right\}$

because $x = 0$, $y = 1$, $r = 1$, $\theta_p = \pi/2$

(a) For $K = 0$, we obtain,

$(i)^{\frac{1}{2}}_{k=0} = \cos\left(\frac{\pi/2 + 0}{2}\right) + i\sin\left(\frac{\pi/2 + 0}{2}\right)$

$= \frac{1}{\sqrt{2}} + i/\sqrt{2}$

$= \frac{1}{\sqrt{2}}(1 + i)$

(b) $K = 1$, we obtain

$(i)^{\frac{1}{2}}_{k=1} = \cos\left(\frac{\pi/2 + 2\pi}{2}\right) + i\sin\left(\frac{\pi/2 + 2\pi}{2}\right)$

$= \cos\pi/4 \cos\pi + i\sin\pi/4 \cos\pi$

$= -\frac{1}{\sqrt{2}} - i/\sqrt{2} = -\frac{1}{\sqrt{2}}(1 + i)$

$\Rightarrow (i)^{1/2}_{k=1} = -\frac{1}{\sqrt{2}}(1 + i)$

Hence roots $(i)^{1/2} = \pm\frac{1}{\sqrt{2}}(1 + i)$ //

$\Delta x \to 0$ $\quad \Delta x + i\Delta y$

$\Delta y \to 0$