

Determination of polynomial when some functional values of the function are given:

8(1)

Ex.

What is the lowest degree polynomial which takes the following values

x	0	1	2	3	4	5
$f(x)$	0	3	8	15	24	35

Solⁿ We have

$$f(a+nh) = f(a) + {}^n C_1 \Delta f(a) + {}^n C_2 \Delta^2 f(a) + \dots + {}^n C_n \Delta^n f(a) \rightarrow (1)$$

Putting $a=0, h=1, n=x$ in (1), we get

$$f(x) = f(0) + {}^x C_1 \Delta f(0) + {}^x C_2 \Delta^2 f(0) + \dots + {}^x C_n \Delta^n f(0)$$

$$= f(0) + x \Delta f(0) + \frac{x(x-1)}{2!} \Delta^2 f(0) + \dots \rightarrow (2)$$

We now construct the following difference table:

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$
0	0	3		
1	3	5	2	0
2	8	7	2	0
3	15	9	2	0
4	24	11	2	
5	35			

$$\left[\because {}^x C_n = \frac{x!}{(x-n)!} \right]$$

$${}^x C_2 = \frac{x!}{2!(x-2)!}$$

$$= \frac{x(x-1)}{2!}$$

Putting these values of $f(0), \Delta f(0), \Delta^2 f(0), \Delta^3 f(0)$ in (2), we get -

$$f(x) = 0 + x \cdot 3 + \frac{x(x-1)}{2!} \cdot 2 + 0 = 3x + x(x-1)$$

$$\Rightarrow f(x) = x^2 + 2x \leftarrow \underline{\underline{\text{Ans.}}}$$

Ex. Find the polynomial of lowest degree which satisfies the following set of numbers
 0, 7, 26, 63, 124, 215, 342, 511.

Solⁿ. First we construct the following diff. table.

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$
0	0				
1	7	7			
2	26	19	12	6	0
3	63	37	18	6	0
4	124	61	24	6	0
5	215	91	30	6	0
6	342	127	36	6	0
7	511	169	42	6	

Now, we know,

$$f(a+nh) = f(a) + {}^n C_1 \Delta f(a) + {}^n C_2 \Delta^2 f(a) + \dots + {}^n C_n \Delta^n f(a) \quad \rightarrow (1)$$

Putting $a = 0, h = 1, n = x$ in (1), we get

$$f(x) = f(0) + {}^x C_1 \Delta f(0) + {}^x C_2 \Delta^2 f(0) + {}^x C_3 \Delta^3 f(0) + \dots$$

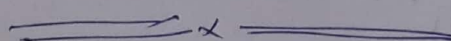
$$\Rightarrow f(x) = 0 + {}^x C_1 \cdot 7 + {}^x C_2 \cdot 12 + {}^x C_3 \cdot 6 + 0 \dots$$

$$= 0 + 7 x^{(1)} + 12 \cdot \frac{x^{(2)}}{2!} + 6 \cdot \frac{x^{(3)}}{3!} + 0 \dots$$

$$= 7x + 6x(x-1) + x(x-1)(x-2)$$

$$= 7x + 6x^2 - 6x + x^3 - 3x^2 + 2x$$

$$\Rightarrow f(x) = x^3 + 3x^2 + 3x \quad \leftarrow \text{Ans.}$$



Ex. Find the function whose first difference is e^x .

Sol. We know that-

$$\begin{aligned} \Delta e^x &= e^{x+h} - e^x, \text{ h being the interval of differencing} \\ &= e^x \cdot e^h - e^x \\ &= e^x (e^h - 1) \end{aligned}$$

$$\begin{aligned} \Rightarrow e^x &= \frac{1}{e^h - 1} \Delta e^x \\ &= \Delta \left[\frac{e^x}{e^h - 1} \right] \end{aligned}$$

i.e. we get,

$$\Delta \left[\frac{e^x}{e^h - 1} \right] = e^x$$

∴ The required function

$$f(x) = \frac{e^x}{e^h - 1} \leftarrow \text{Ans.}$$

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