Tensor

Lect – I

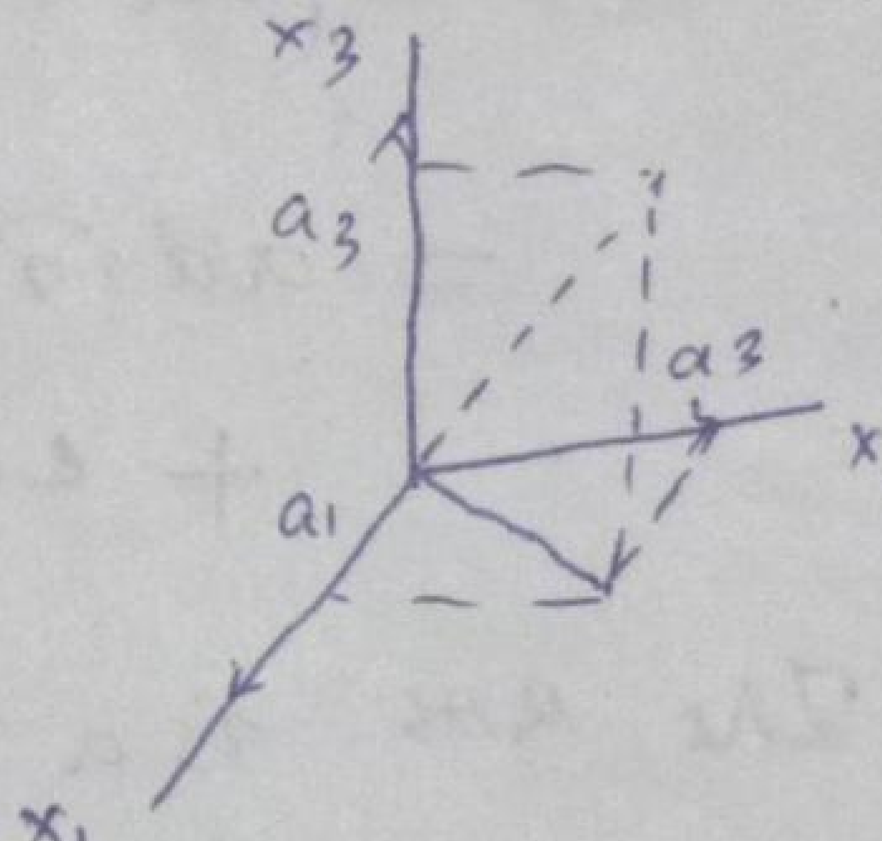
Before going definition of tensor, we should know the concept about —

(a) Scalar (α) = A physical quantity of which its component is independent of the choice of the coordinate system. eg → temp^r, density, pot^n etc.

(b) Vector (a) = The physical quantity of which its components ~~that shows~~ is dependent of the choice of the co-ordinate ~~sy~~ system. eg → velocity, force heat flow → etc)

(c) Tensor (A) → A tensor contains the information about the directions and the directions in those direction.

In cartesian co-ordinate, a vector can be expressed by three order scales



$$a = a_1e_1 + a_2e_2 + a_3e_3$$
$$= \sum_{i=1}^{3} a_i e_i$$

① Dot Product of Inner Product

$$a \cdot b = |a||b|\cos(a,b)$$

Here three basis vectors of cartesian co-ordinate

$$e_i \cdot e_j = \begin{cases} 1 & \text{if } i=j \\ 0 & \text{if } i\neq j \end{cases} = \delta_{ij} \rightarrow \text{Kronecker delta}$$

② Cross Product

$$a\times b = \begin{vmatrix} e_1 & e_2 & e_3 \\ a_1 & a_2 & a_2 \\ b_1 & b_2 & b_3 \end{vmatrix} \Rightarrow$$ Product new vector having the direction "up" & down

$$\varepsilon_{ijk} = \begin{cases} 1 \rightarrow & \text{even Permutation} \\ -1 - & \text{odd Permutation} \\ 0 - & \text{repeated index} \end{cases}$$

(d) <u>Dyads and other higher order Product.</u>

Let multiply the two quantities
$(a+b+c)$ & $(d+e+f)$

$\Rightarrow (a+b+c)(d+e+f) = ab + ae + af + bd + be + bf + cd + ce + cf$

this product has $= 3^2 = 9$ terms

lly $\rightarrow$

$A = ai + bj + ck$, $B = di + ej + fk$

$AB = (ai + bj + ck)(di + ej + fk)$

$= adii + aeij + afik + bdji + bejj + bfjk + cdki + cekj + cfkk$

The RHS is a new entity that having following meaning $\rightarrow$

$\Rightarrow$ Name this entity as dyad or dyadic (meaning di or double,

Let we take dot product or inner product A & D generates a scalar quantity. But if we take cross product constitute new dyand, another vector.
Thus the dyand an unknown entity which enter in the Mathematical world.

Now to check the dyad as follows the doesn't follow the Commutative law -

Proof

Taking inner product with another vector X, to AD dyad.

ie $X \cdot AD = (X \cdot A) D$, $\xi = X \cdot A$ scalar

$= \xi D$

Again $AD \cdot X = A(D \cdot X) = A(D \cdot X)$

$= \psi A$

ξD is new. vector whose direction to the D

&

ψA " " " " " A

So, $X \cdot AD \neq AD \cdot X$

Thus the dyad define as

" A dyad is any quantity that operates on a vector through the inner product produces a new vector with different magnitude and direction from the origin"

Dyadic Arithmetic **