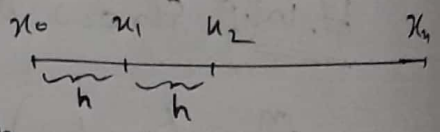
Difference Table.

(1) Forward or descending difference table:

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$
a	$f(a)$				
		$\Delta f(a)$			
$a+h$	$f(a+h)$		$\Delta^2 f(a)$		
		$\Delta f(a+h)$		$\Delta^3 f(a)$	
$a+2h$	$f(a+2h)$		$\Delta^2 f(a+h)$		$\Delta^4 f(a)$
		$\Delta f(a+2h)$		$\Delta^3 f(a+h)$	
$a+3h$	$f(a+3h)$		$\Delta^2 f(a+2h)$		
		$\Delta f(a+3h)$			
$a+4h$	$f(a+4h)$				

OR

Let $y = f(x)$.

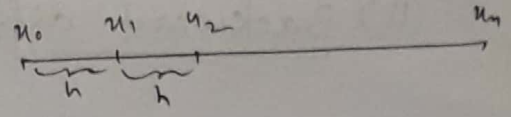


$\therefore y_s = f(x_s)$, $x_s = x_0 + sh$

then $\Delta y_s = y_{s+1} - y_s$, $\Delta^2 y_s = \Delta(\Delta y_s) = \Delta y_{s+1} - \Delta y_s$

x	$y=f(x)$	Δy	$\Delta^2 y$	$\Delta^3 y$
$a=x_0$	y_0			
		$\Delta y_0 = y_1 - y_0$		
$a+h=x_1$	y_1		$\Delta^2 y_0 = \Delta y_1 - \Delta y_0$	
		$\Delta y_1 = y_2 - y_1$		$\Delta^3 y_0 = \Delta^2 y_1 - \Delta^2 y_0$
$a+2h=x_2$	y_2		$\Delta^2 y_1 = \Delta y_2 - \Delta y_1$	
		$\Delta y_2 = y_3 - y_2$		
$a+3h=x_3$	y_3			

(ii) Backward or ascending difference Table:



Let $y = f(x)$

$\therefore y_s = f(x_s)$, $x_s = x_0 + sh$

$\therefore \nabla y_s = y_s - y_{s-1}$

x	y	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$
x_0	y_0				
		$\nabla y_1 = y_1 - y_0$			
x_1	y_1		$\nabla^2 y_2 = \nabla y_2 - \nabla y_1$		
		$\nabla y_2 = y_2 - y_1$		$\nabla^3 y_3 = \nabla^2 y_3 - \nabla^2 y_2$	
x_2	y_2		$\nabla^2 y_3 = \nabla y_3 - \nabla y_2$		$\nabla^4 y_4 = \nabla^3 y_4 - \nabla^3 y_3$
		$\nabla y_3 = y_3 - y_2$		$\nabla^3 y_4 = \nabla^2 y_4 - \nabla^2 y_3$	
x_3	y_3		$\nabla^2 y_4 = \nabla y_4 - \nabla y_3$		
		$\nabla y_4 = y_4 - y_3$			
x_4	y_4				

Ex. Construct a forward difference table and a backward difference table for the following:

x:	0	1	2	3	4
$f(x)$:	20	25	32	36	40

Sol: ① forward difference table:

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$
0	20				
		5			
1	25		2		
		7		−5	
2	32		−3		8
		4		3	
3	36		0		
		4			
4	40				

(ii) Backward difference Table :

x	$f(x)$	$\nabla f(x)$	$\nabla^2 f(x)$	$\nabla^3 f(x)$	$\nabla^4 f(x)$
0	20				
		-5			
1	25		2		
		-7		5	
2	32		-3		8
		-4		-3	
3	36		0		
		-4			
4	40				

Ex. Write down the difference table for $y = 3x^3$, where x takes the integral values 1, 2, 3, 4.

Sol^n

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	
1	1				
		7			
2	8		12		
		19		6	
3	27		18		
		37			
4	64				