Relation between Δ & E :→

We have,

$$\Delta f(x) = f(x+h) - f(x)$$

& $$E f(x) = f(x+h)$$

$$\therefore \Delta f(x) = E f(x) - f(x)$$

$$\Rightarrow f(x) + \Delta f(x) = E f(x)$$

$$\Rightarrow (1+\Delta) f(x) = E f(x)$$

Since this is true for any function $f(x)$, so

$$\boxed{1+\Delta = E}$$

Here 1 is not ordinary number, it is the unit operator, which keeps the function as it is.

//

Scanned with CamScanner

§. We have

$$E f(x) = f(x+h)$$

$$E^2 f(x) = E[E f(x)]$$

$$= E f(x+h)$$

$$= f(x+2h)$$

Similarly,

$$E^3 f(x) = f(x+3h).$$

\- - - - - -

In general,

$$E^n f(x) = f(x+nh)$$

Also;

$$E^{-n} f(x) = f(x-nh)$$

$$E^{-1} f(x) = f(x-h).$$

---

§. Relation between ∇ and E.

We have, $\nabla f(x) = f(x) - f(x-h)$

$$= f(x) - E^{-1} f(x)$$

$$= (1 - E^{-1}) f(x)$$

Since this is true for any function $f(x)$, so

$$\boxed{\nabla \equiv 1 - E^{-1}}$$

//

Ex. If Δ and ∇ be the first descending difference operator and first ascending difference operator respectively of a function $f(x)$, show that $(\Delta - \nabla) \equiv \Delta\nabla$.

Solution: By definition, we have

$$\Delta f(x) = f(x+h) - f(x)$$

and $\nabla f(x) = f(x) - f(x-h)$,

where h is the interval of differencing.

Now,

$$\begin{aligned}
\Delta\nabla f(x) &= \Delta[\nabla f(x)] \\
&= \Delta[f(x) - f(x-h)] \\
&= \Delta f(x) - \Delta f(x-h) \\
&= \Delta f(x) - [f(x) - f(x-h)] \\
&= \Delta f(x) - \nabla f(x) \\
&= (\Delta - \nabla) f(x)
\end{aligned}$$

Since this is true for any function $f(x)$, so

$$\boxed{\Delta\nabla \equiv \Delta - \nabla}$$

//

Scanned with CamScanner

Ex. Prove that $\Delta E = E\Delta$

Solution: We have

$$(\Delta E) f(x) = \Delta (E f(x))$$
$$= \Delta f(x+h)$$
$$= f(x+2h) - f(x+h) \rightarrow (i)$$

Again,

$$(E \Delta) f(x) = E(\Delta f(x))$$
$$= E(f(x+h) - f(x))$$
$$= E f(x+h) - E f(x)$$
$$= f(x+2h) - f(x+h) \rightarrow (ii)$$

$\therefore$ (i) & (ii) $\Rightarrow$

$$(\Delta E) f(x) = (E \Delta) f(x)$$

But this is true for any function $f(x)$, so

$$\Delta E \equiv E\Delta.$$

//

Some properties of Δ and E :

1. $\Delta c = 0$, where c is a constant.

2. $\Delta c = c\Delta$

3. (i) $\Delta[f(x)+g(x)+\cdots] = \Delta f(x) + \Delta g(x) + \cdots$

(ii) $E[f(x)+g(x)+\cdots] = E f(x) + E g(x) + \cdots$

4. i) $\Delta[cf(x)] = c\Delta f(x)$

ii) $E[cf(x)] = c E f(x)$

5. i) $\Delta^p(\Delta^q f(x)) = \Delta^{p+q} f(x)$

ii) $E^p(E^q f(x)) = E^{p+q} f(x)$

* 6. $E(\Delta f(x)) = \Delta[E f(x)]$

(7) i) $\Delta(f(x)\cdot g(x)) \neq f(x)\Delta g(x)$

ii) $E[f(x)\cdot g(x)] \neq f(x) E g(x)$

(8) $\Delta^m[\Delta^{-m} f(x)] = f(x)$

$E^m[E^{-m} f(x)] = f(x)$

Scanned with CamScanner