S-VI/2

Form No. Ex. 10

Invigilator

Date

Sl. No. V E 062165

ADDITIONAL SHEET
HIGHER SECONDARY FINAL EXAMINATION, 20

Roll .... No. .... Subject .... Paper ....

Registration No. .... Year .... Stream ....

Complex variable

The quadratic equation

$$ax^2+bx+c=0.$$

Solutions are

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

When $4ac > b^2$, we write the complex solution

$$x = \frac{-b \pm i\sqrt{4ac-b}}{2a} \quad \text{where } i = \sqrt{-1}$$

A complex variable, $z = x+iy$, is an ordered pair of real variable x and y which satisfies certain laws of operation. The real and imaginary parts of z are $\mathrm{Re}\,z = x$ & $\mathrm{Im}\,z = y$.

Algebraic operation :-

(1) Addition & Substraction

$$z_1 \pm z_2 = (x_1 \pm x_2) + i(y_1 \pm y_2)$$

(2) Multiplication : $(z_1 z_2) = (x_1+iy_1)(x_2+iy_2)$

$$= (x_1x_2 - y_1y_2) + i(x_1y_2 + x_2y_1)$$

Scanned by TapScanner

x_2

(3) Division :-
$$\frac{z_1}{z_2} = \frac{x_1+iy_1}{x_2+iy_2} = \frac{(x_1+iy_1)(x_2-iy_2)}{(x_2+iy_2)(x_2-iy_2)}$$

$$\therefore \frac{z_1}{z_2} = \frac{x_1x_2+y_1y_2}{x_2^2+y_2^2} + i\left(\frac{x_2y_1-x_1y_2}{x_2^2+y_2^2}\right)$$

(4) Associative ~~of~~ law of addition :-

$$z_1+(z_2+z_3) = (z_1+z_2)+z_3$$

(5) Commutative law of multiplication :-

$$z_1z_2 = z_2z_1$$

(6) Associative law of multiplication

$$z_1(z_2z_3) = (z_1z_2)z_3$$

(7) Distributive law

$$z_1(z_2+z_3) = z_1z_2+z_1z_3$$

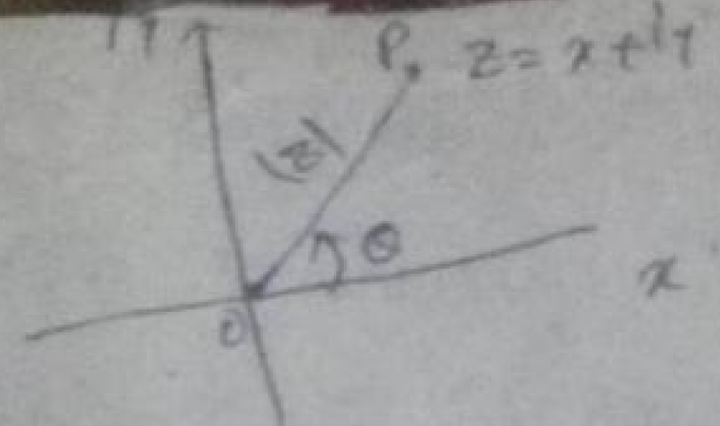
Argand diagram : Vector Representation :-

Since in a complex number $x+iy$ can be consider as an ordered pair of real number (x,y), can be represented such numbers by points in an xy plane which called the complex plane or Argand plane as simple a vector representation for $(z=x+iy)$ ~~where~~. The y-axis is an imaginary axis and x-axis is real axis. The idea profounded by Argand (1768-1822)

Scanned by TapScanner

complex number represented by P,

x and to each complex number there corresponding one and only one point in the plane, and conversely to each point in the plane there corresponds one and only one complex number. Because of this we often refer to the complex number z as the point z.



P-

Part

Sometimes we refer to the the x and y axis as the real and ~~imam~~ imaginary axes respectively and to the complex plane as the z plane. The distance between two points (suppose)

$$z_1 = x_1 + iy_1 \quad \& \quad z_2 = x_2 + iy_2$$ in the z plane

as $$|z_1 - z_2| = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

(b) Polar form of complex number :-

From the figure

$$x = r\cos\theta, \quad y = r\sin\theta$$

$$\therefore z = r\cos\theta + ir\sin\theta$$

$$= r(\cos\theta + i\sin\theta) \quad \text{— (1)}$$

Now $r^2 = x^2 + y^2$

$$\therefore r = \sqrt{x^2 + y^2} \quad \text{— (2)}$$ (always real value)

Scanned by TapScanner

The equation (2) represent the modulus or absolute value of z represence as $|z| = \sqrt{x^2+y^2}$ and eqn (1) is called the polar form of complex number. The θ denotes the argument (phase) of z represence as $\theta = \operatorname{Arg} z$. Due to non unique value of Argument of z ie determined up to a multiple of 2π, Hence The Argument

$$\theta = \theta_p \pm 2n\pi, \quad \text{Where } n = 0, \pm 1, \pm 2, \ldots)$$

$\theta_p \rightarrow$ The principal argument.

Problem Express each of the following complex numbers to polar form

(a) $2+2\sqrt{3}i$, $\quad r = \sqrt{4+12} = 4$

$$\operatorname{Arg} \theta = \sin^{-1}\left(\frac{2\sqrt{3}}{4}\right) = 60^\circ = \pi/3 \text{ (radian)}$$

$$\therefore 2+2\sqrt{3}i = 4(\cos \pi/3 + i \sin \pi/3)$$

(b) $-5+5i$, $\quad r = 5\sqrt{2} \quad \theta = 180-45 = 135^\circ = 3\pi/4$ radian

c) $-\sqrt{6} - i\sqrt{2} \quad r = 2\sqrt{2}, \quad \theta = 180 + 30^\circ = 210 = 7\pi/6$ radian

(d) $-3i \quad r = 3, \quad \theta = 270 = 3\pi/2$ radian

Scanned by TapScanner

Complex Conjugate :-

If $z = x + iy$ be a complex number and its complex defined as

$$z^* = \bar{z} = x - iy$$ its graphical presentation

is shown in the figure at where changing the sign of imaginary part only.

In this context

① $$z\bar{z} = (x+iy)(x-iy) = x^2+y^2 = r^2$$

$$\therefore r = \sqrt{x^2+z^2} = |z|^2$$

② $$\overline{(z_1+z_2)} = \bar{z}_1 + \bar{z}_2$$

③ $$\overline{(z_1 z_2)} = \bar{z}_1 \bar{z}_2$$

④ $$\overline{\left(\frac{z_1}{z_2}\right)} = \frac{\bar{z}_1}{\bar{z}_2}$$

Prob By using of an algebraic analysis, show that

$$|z_1 + z_2| \leq |z_1| + |z_2|$$

Soln The square of the left-hand side of this inequality is

$$|z_1+z_2|^2 = (z_1+z_2)(z_1^* + z_2^*)$$

$$= z_1 z_1^* + z_1 z_2^* + z_1^* z_2 + z_2 z_2^*$$

But $$z_1 z_2^* + z_1^* z_2 \leq 2|z_1 z_2^*| = 2|z_1| \cdot |z_2|$$

$\Delta x \to 0$

$\Delta y \to 0$

Scanned by TapScanner

Scanned by TapScanner