

Differential Equations

(1)

Defⁿ: An equation which involves differentials or differential co-efficients is called a differential eqn.

e.g.

(i) $dx + dy = 0$

(ii) $x dx + y dy = 0$

(iii) $\frac{dy}{dx} + 2y = 3x$

(iv) $\frac{d^2y}{dx^2} + 5 \frac{dy}{dx} + 6y = \log x$ etc. etc.

(v) $(\frac{dy}{dx})^2 + 2y^2 = 4(\frac{dy}{dx}) + 4x$

Order and degree of a differential eqn.

The highest order differential or differential co-efficients present in a differential eqn. is called the order of the differential eqn. Thus the eqn (i), (ii) & (iii) in the above examples are of order 1 (one) while the diff. eqn. (iv) is of 2nd order.

The degree of a differential eqn is the power (or degree) of the highest order differential co-efficient when the equation has been made rational. Thus eqns (i), (ii), (iii) and (iv) are all of first order and eqn (v) is of second order.

Mathematical Models: A mathematical model is a description of a system or a natural law or phenomenon using mathematical concepts and language. The process of developing a mathematical model is called mathematical modeling. OR

A mathematical model usually describes a system by a set of variables and a set of equations that establish relationships between the variables. The actual model is the set of functions that describe the relations between the different variables.

Mathematical models are used in the natural sciences and engineering disciplines, as well as social sciences.

In general, mathematical models are expressed in the form of a differential equation.

Solution of a Differential Eqn:

i) General Solⁿ The relation containing n arbitrary constants which satisfies an ordinary differential eqn. of n^{th} order is called its complete primitive or general solution.

(ii) Particular solution or particular solⁿ of a differential eqn. is one obtained from the complete primitive by assigning definite values of the arbitrary constants.

Formation of Differential Eqn:

By differentiation and eliminating n arbitrary constants from an equation in x and y , we get a differential eqn. of n^{th} order.

H.W. Example: Form a differential from the relⁿ

(i) $y = ax + by^2$

(ii) $c(y+c)^2 = x^3$

Note: Number of arbitrary constant is equal to the order of the differential eqn.